

The Demographic Transition

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Economic Growth and Comparative Development

Phases of Development: Standard of Living

- The Malthusian Epoch

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- The Malthusian Epoch
- The Post-Malthusian Regime

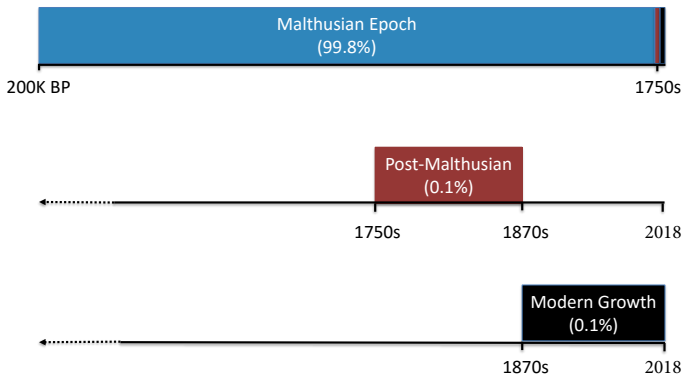
Phases of Development: Standard of Living

- The Malthusian Epoch
- The Post-Malthusian Regime
- The Modern Growth Regime

Phases of Development: Timeline of the Most Developed Economies



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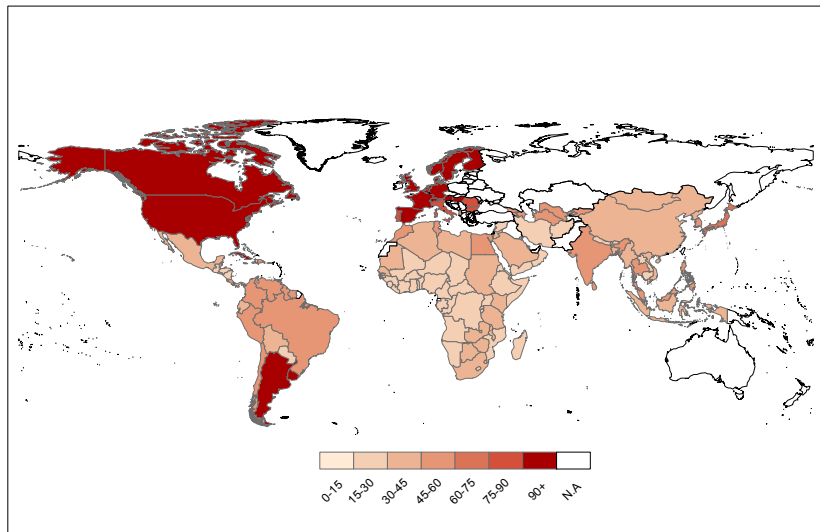
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- Transition to Modern Growth

Variation in years elapsed since the Onset of the Fertility Decline



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⇒ population growth excluding migration

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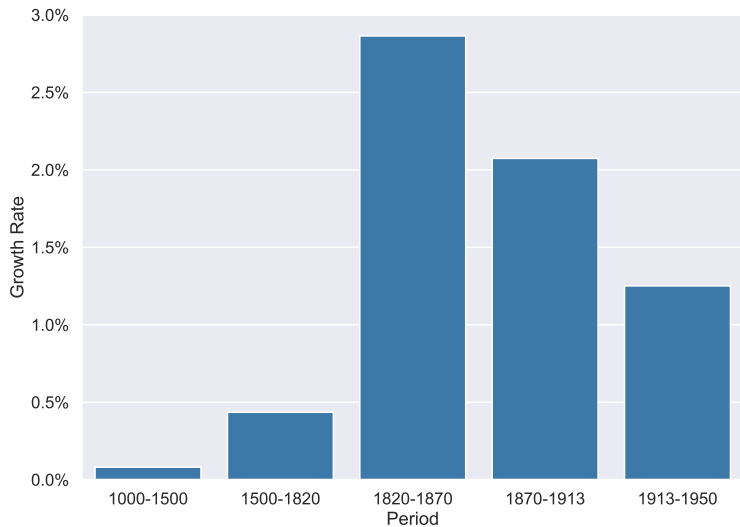
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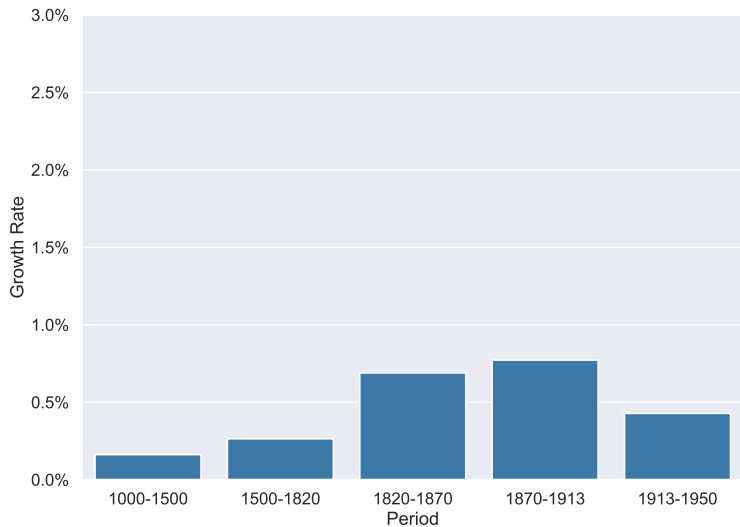
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- TFR & NRR are synthetic rates

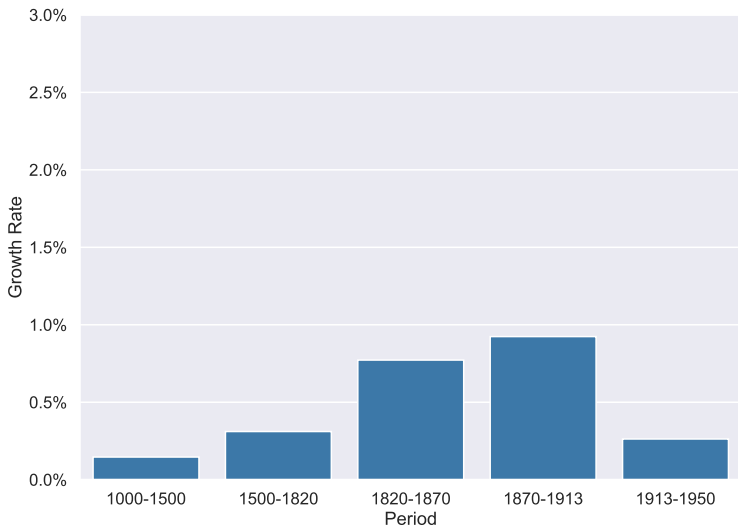
Early Fertility Decline – Western Offshoots



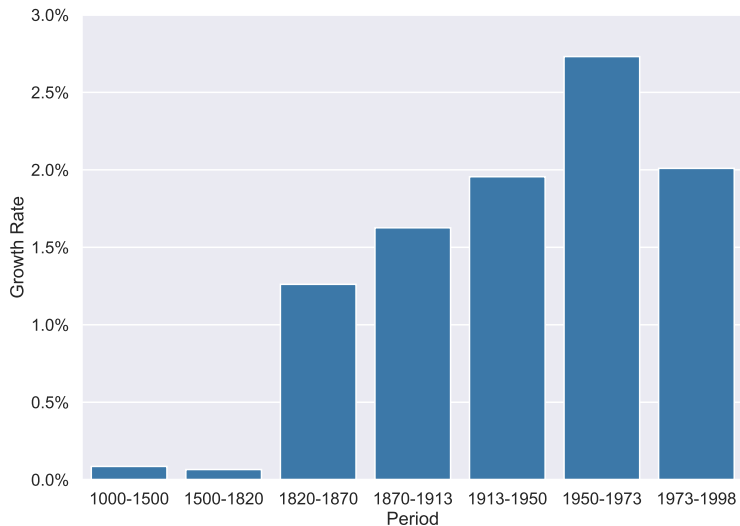
Early Fertility Decline – Western Europe



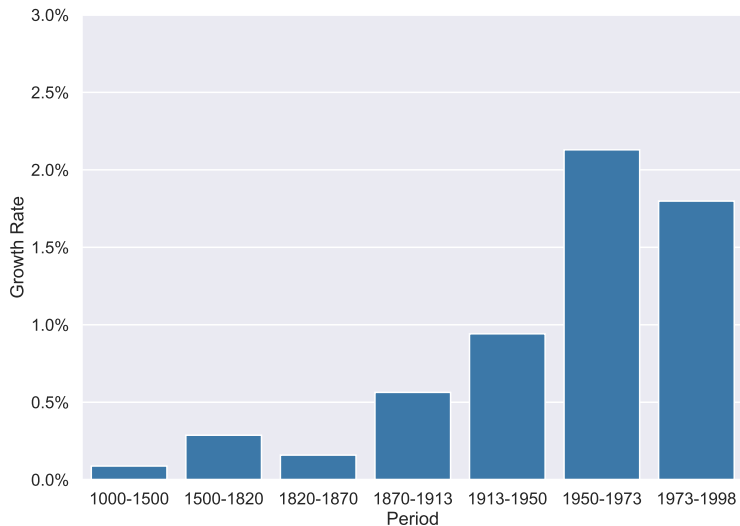
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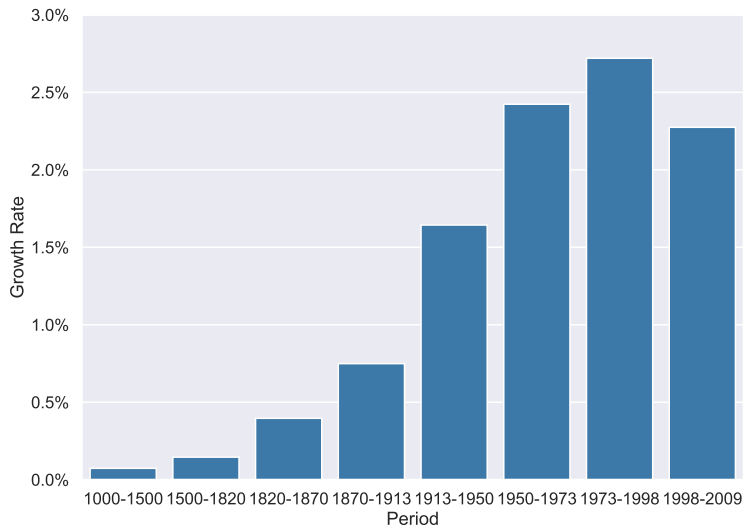
Late Fertility Decline – Latin America



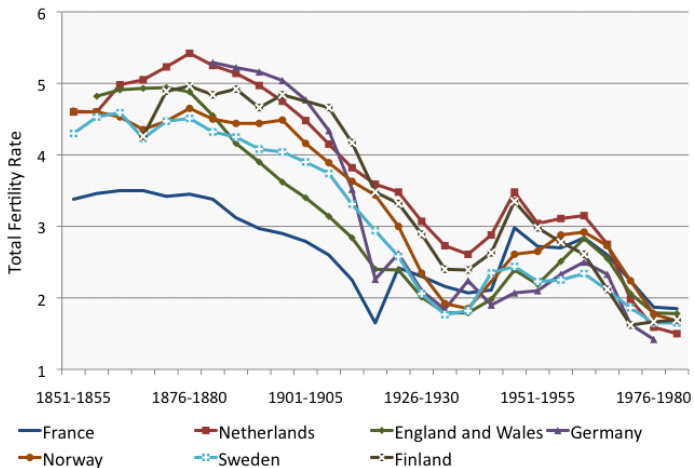
Late Fertility Decline – Asia



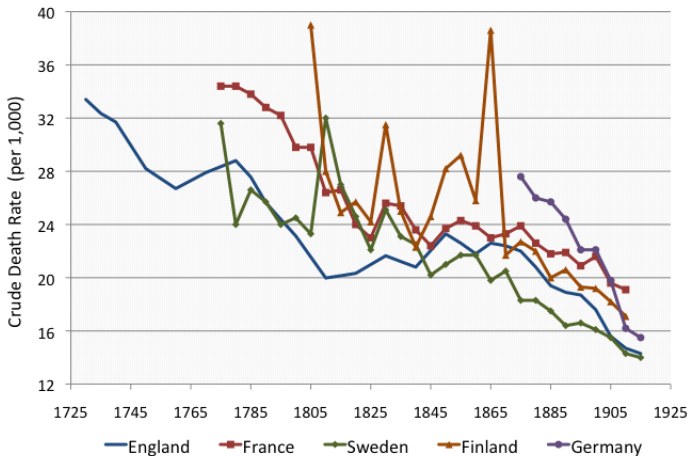
Late Fertility Decline – Africa



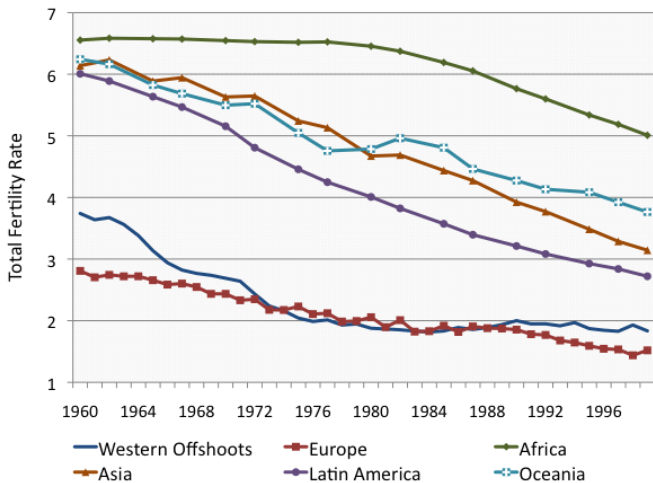
The Demographic Transition in Western Europe: Total Fertility Rates



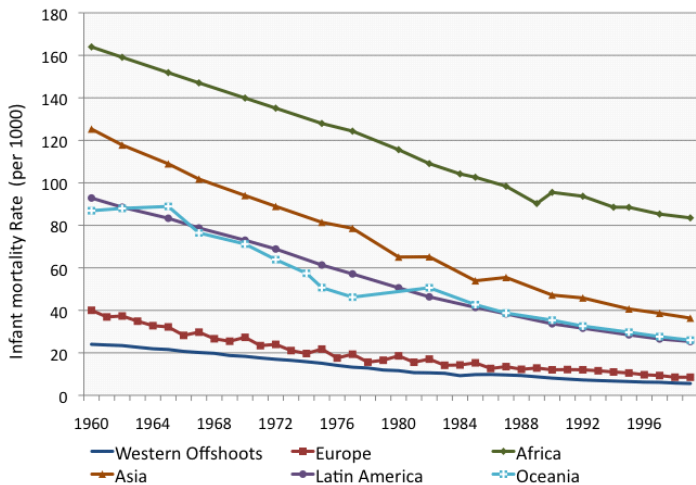
Mortality Decline Western Europe: 1730-1920



The Evolution of Total Fertility Rate across Regions, 1960-1999



Decline in infant mortality rates across regions, 1960-1999



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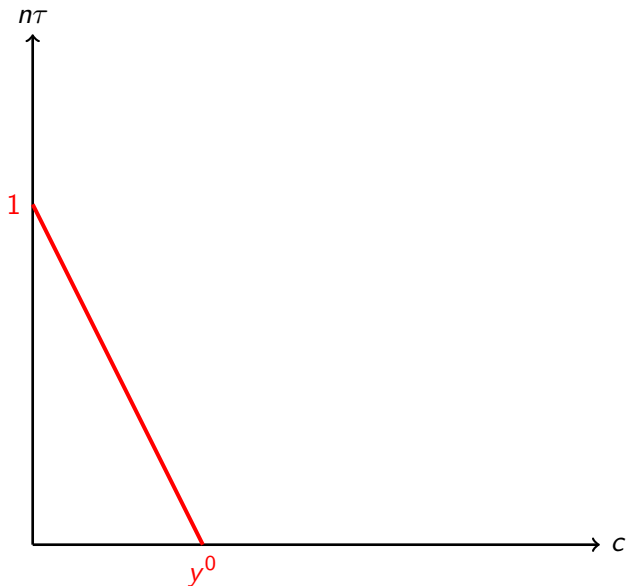
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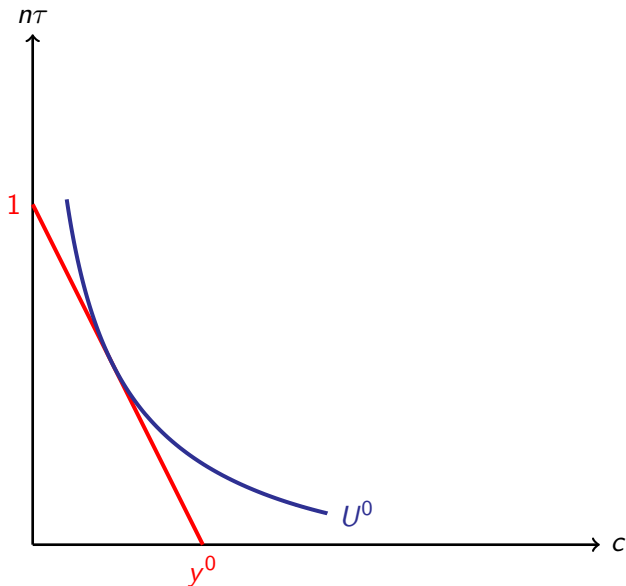
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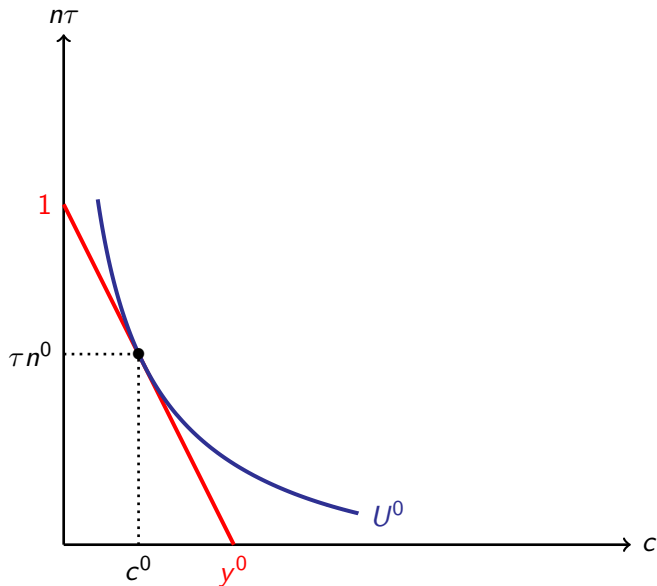
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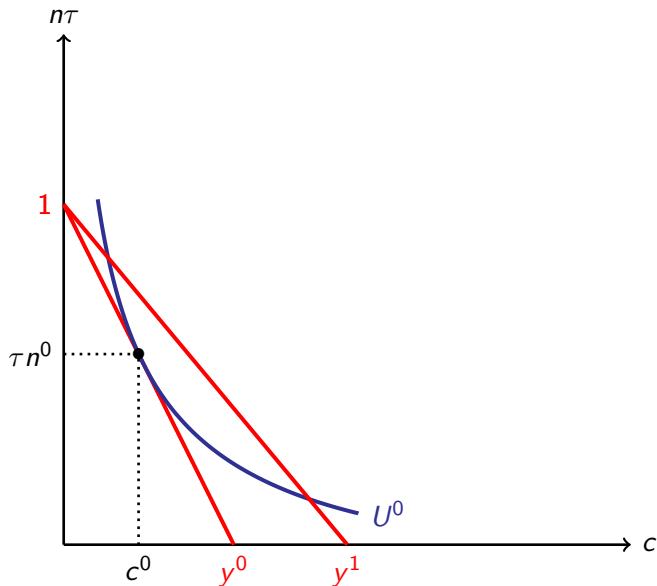
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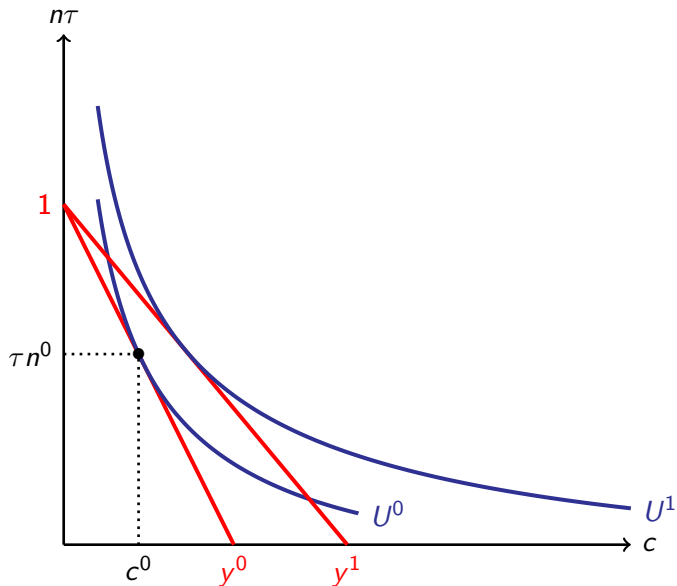
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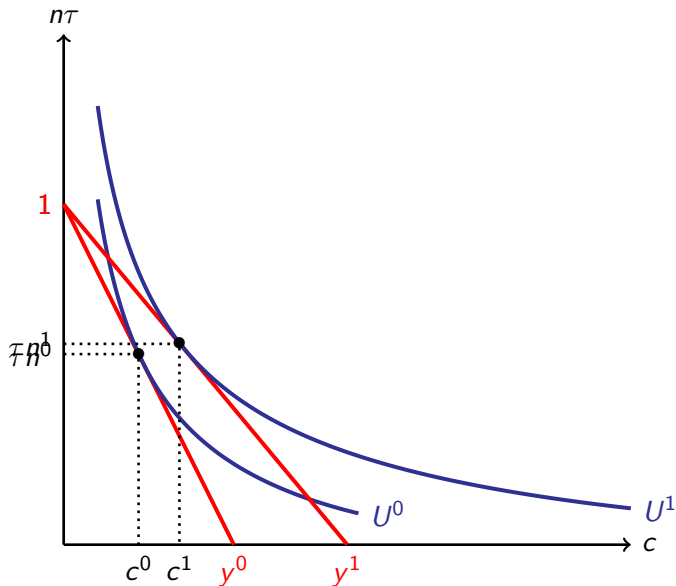
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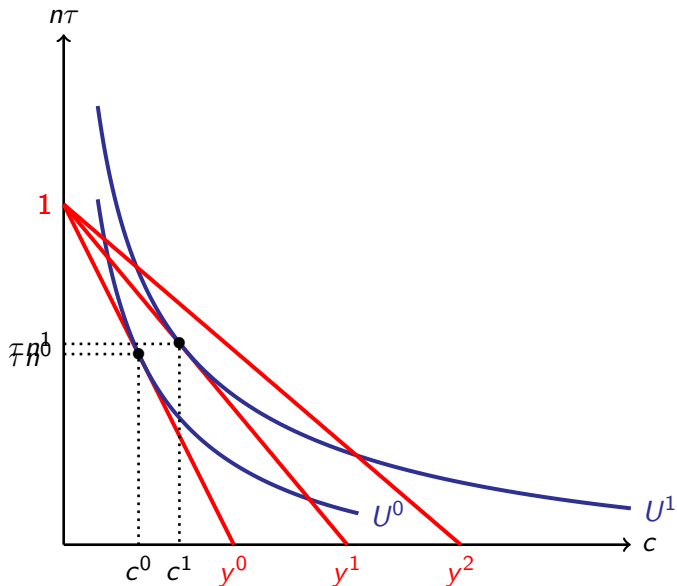
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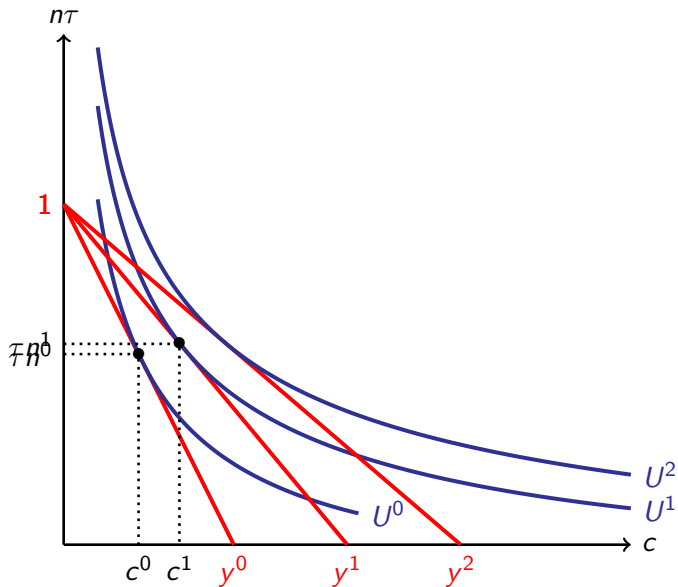
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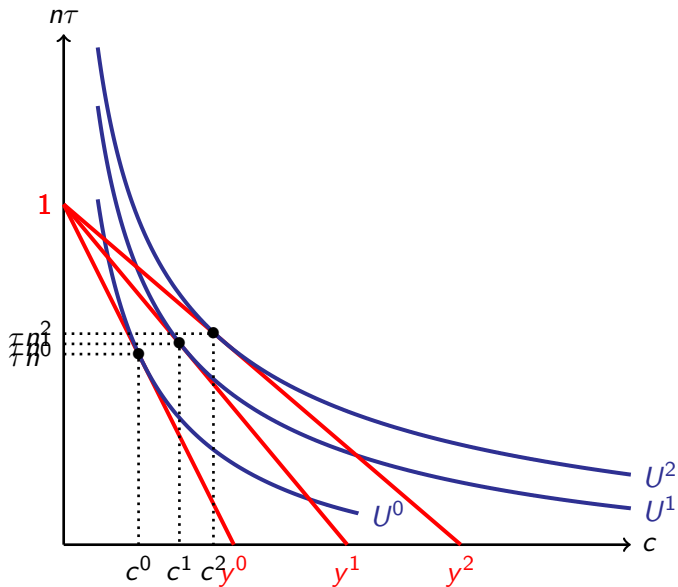
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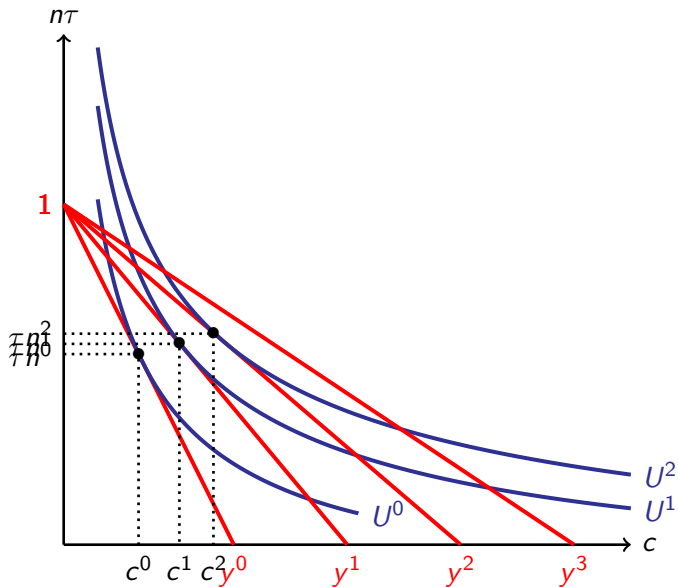
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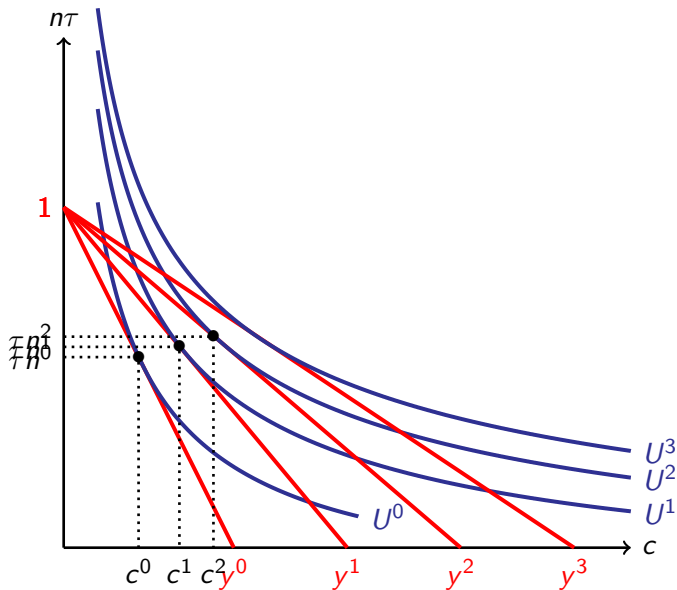
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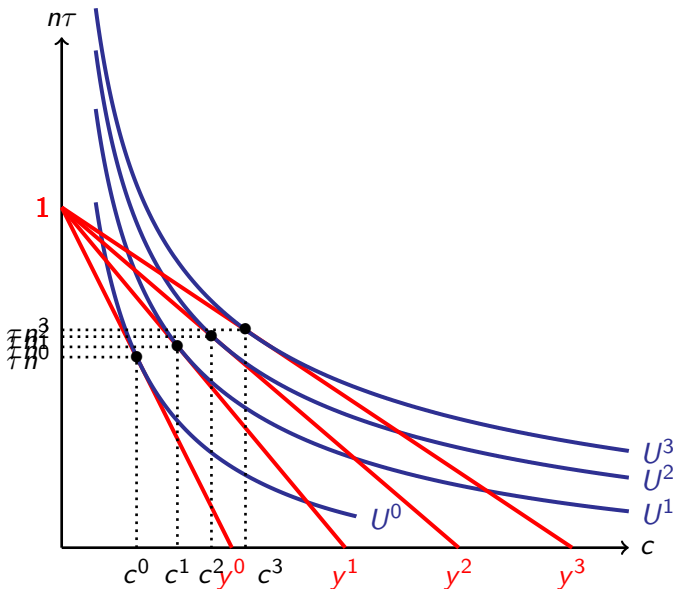
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- The substitution effect dominates at a higher level of income
- As income increases fertility declines
- Fertility declines in the process of development

The Rise in Income - Theoretical Evaluation

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- Preference-based theory (unattractive)
 - Innate bias against child quantity beyond a certain level of income - non-refutable
 - Non-robust (e.g., the class of homothetic preferences will not trigger a fertility decline)

The Rise in Income - Homothetic Preferences

- Preferences:

$$u = n^\gamma c^{(1-\gamma)}$$

$$0 < \gamma < 1$$

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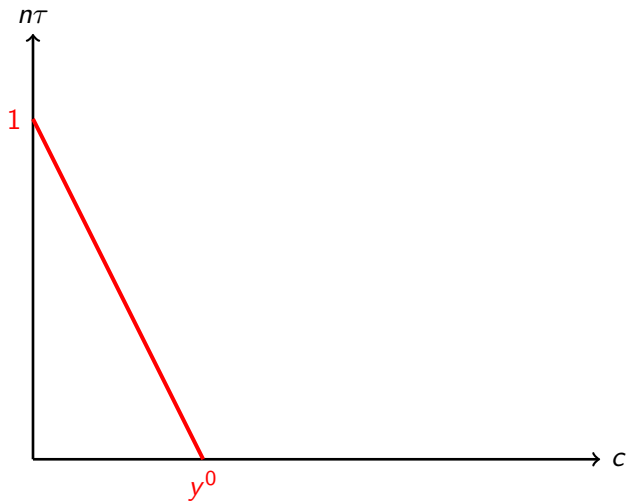
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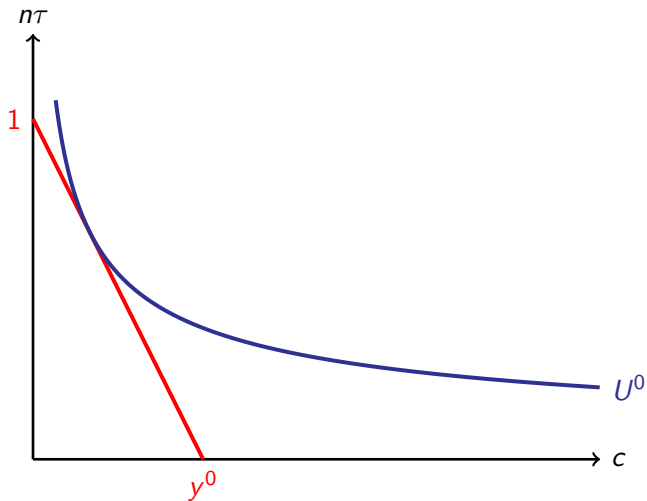
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- Fertility is unaffected by the process of development

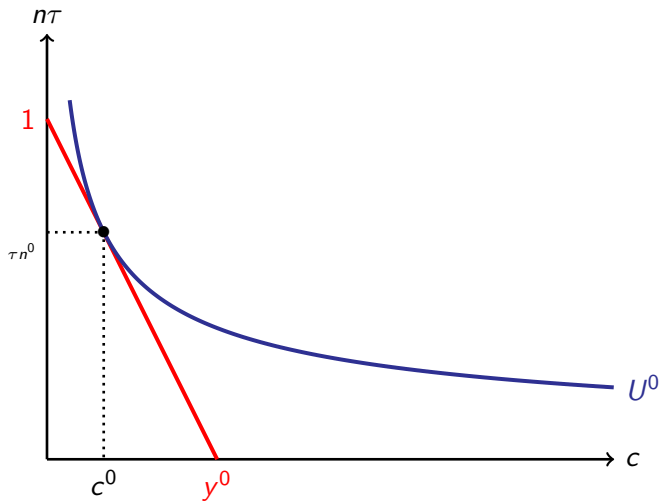
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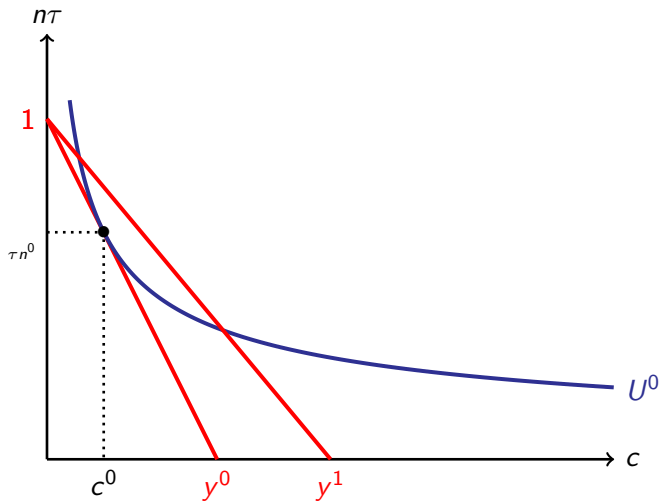
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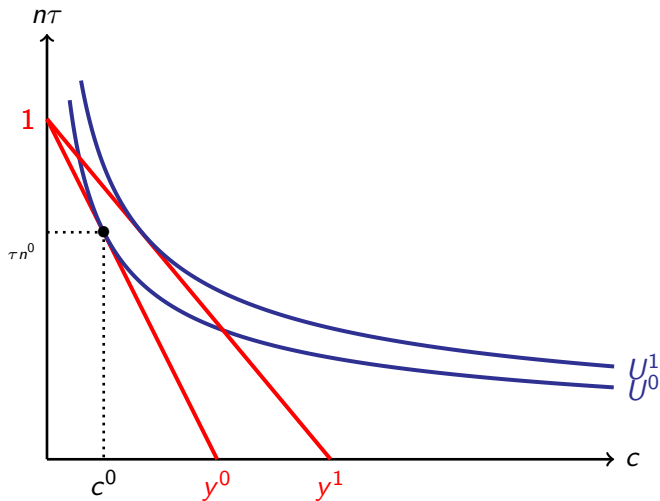
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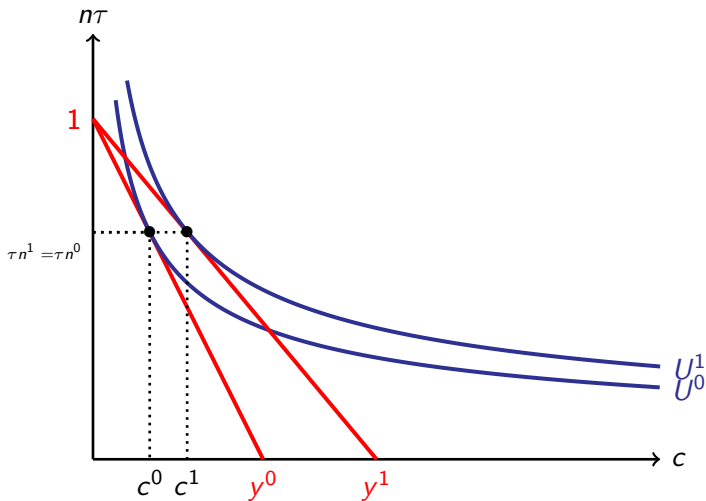
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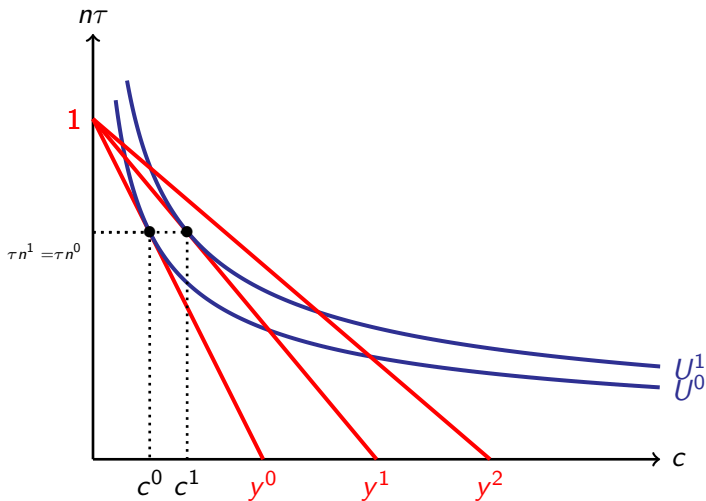
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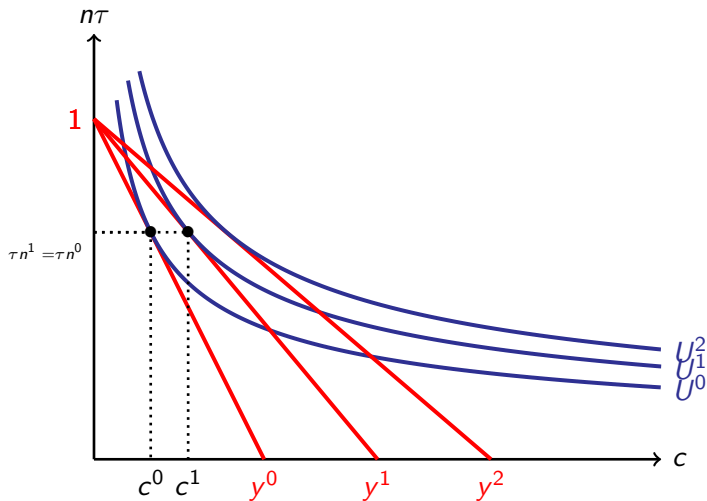
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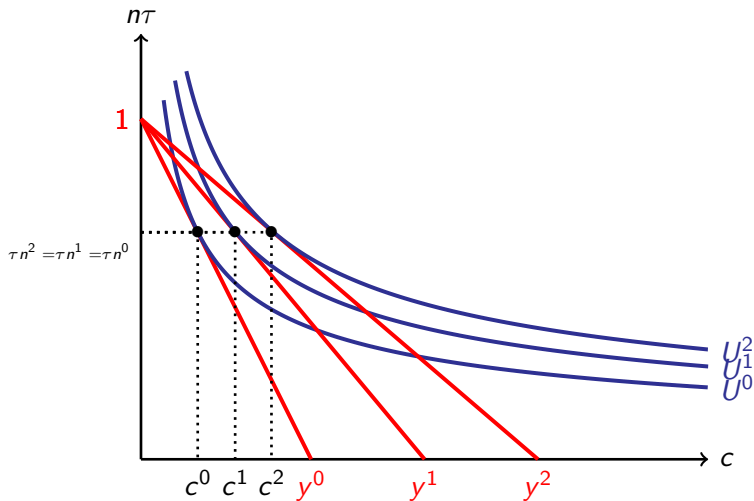
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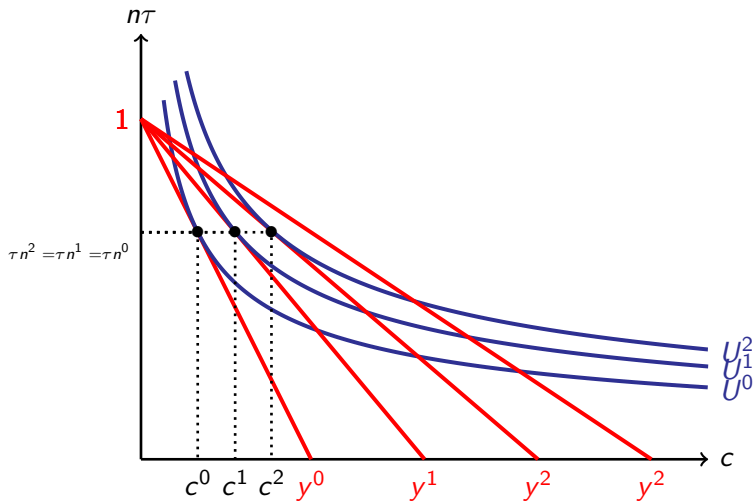
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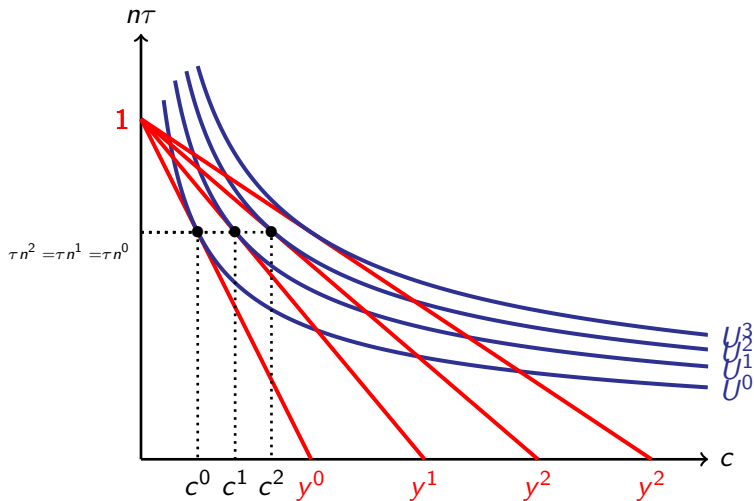
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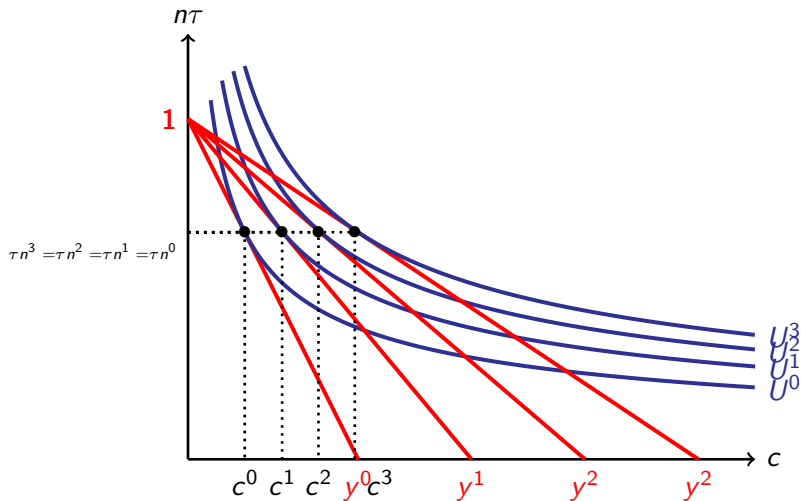
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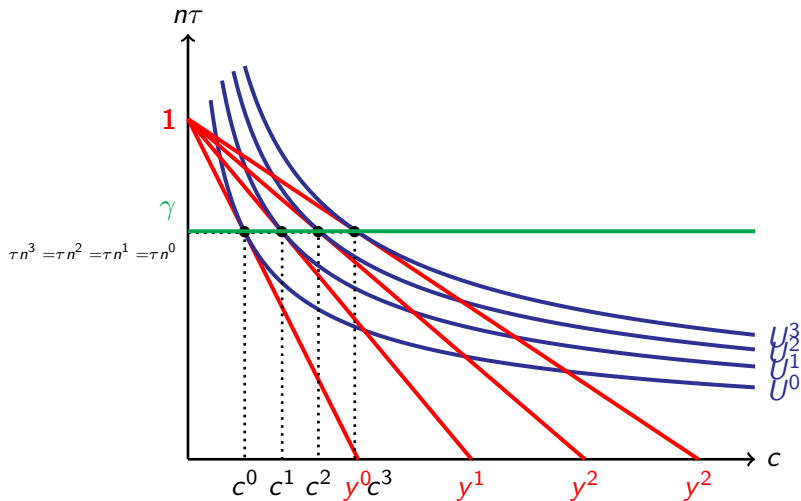
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The Rise in Income: Testable predictions

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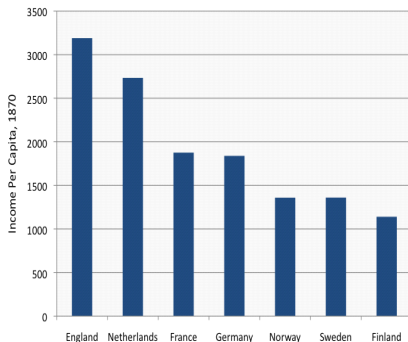
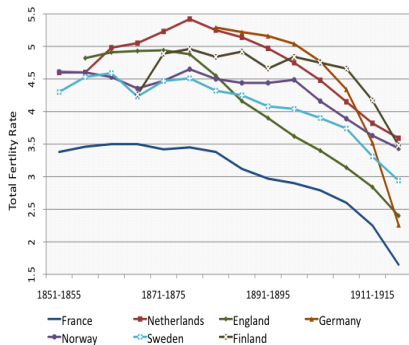
The Rise in Income: Refuting Cross Country Evidence

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Simultaneous DT across European Countries that Differ in Income per Capita



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- Optimal fertility (# of successful pregnancies - TFR)

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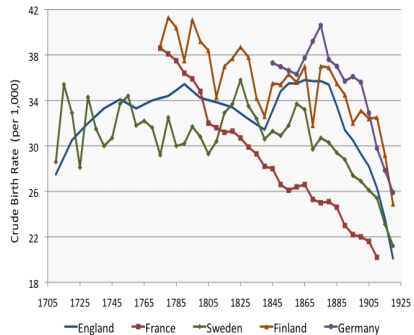
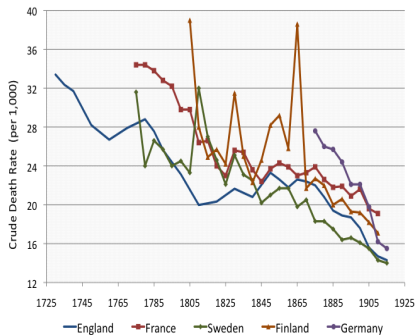
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The Decline in Mortality and Fertility - Evidence



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- Prior to the demographic transition, richer individuals who presumably had better access to financial markets, had larger number of surviving children

The Decline in the Gender Wage Gap

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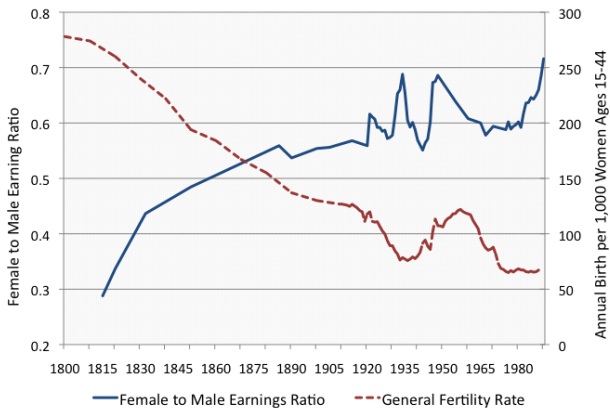
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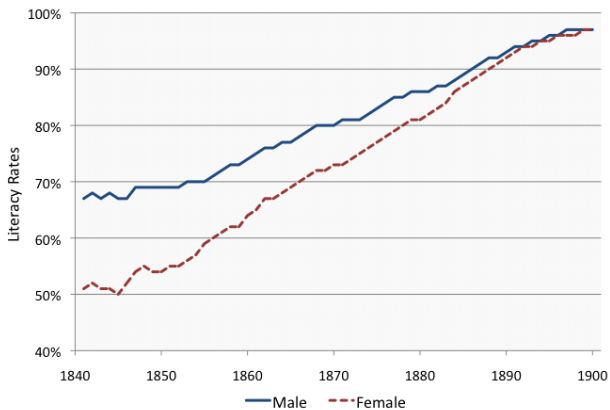
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Evolution of the Gender Earnings Ratio - US



Evolution of the Gender Literacy Gap - England



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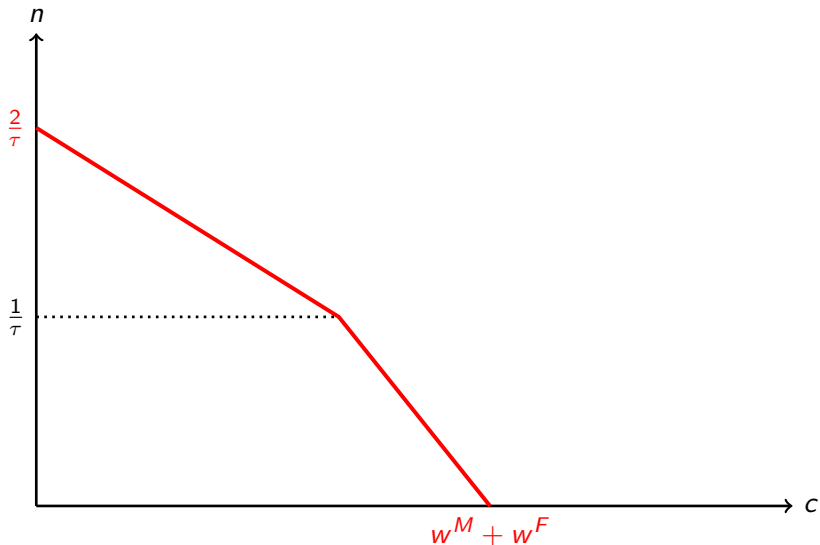
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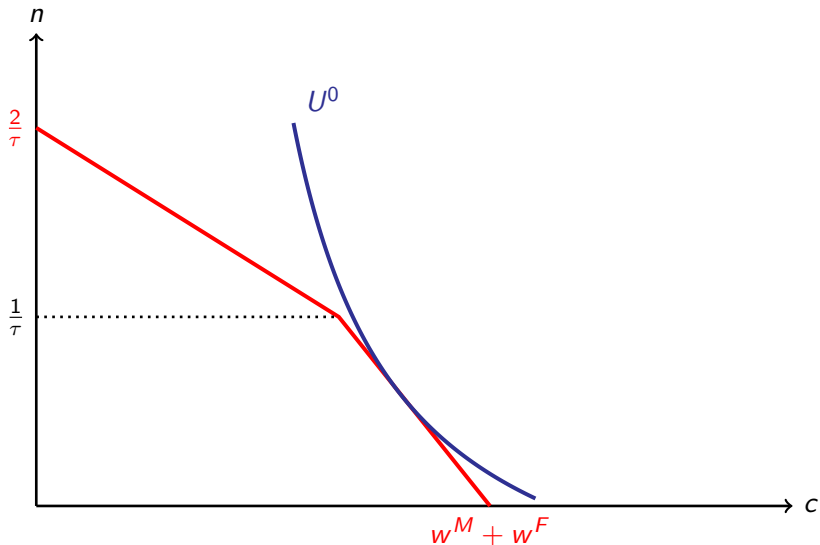
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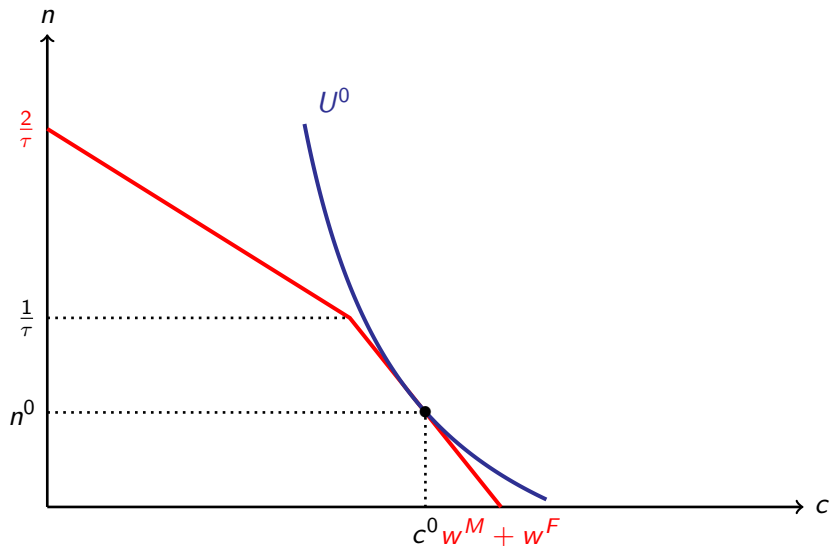
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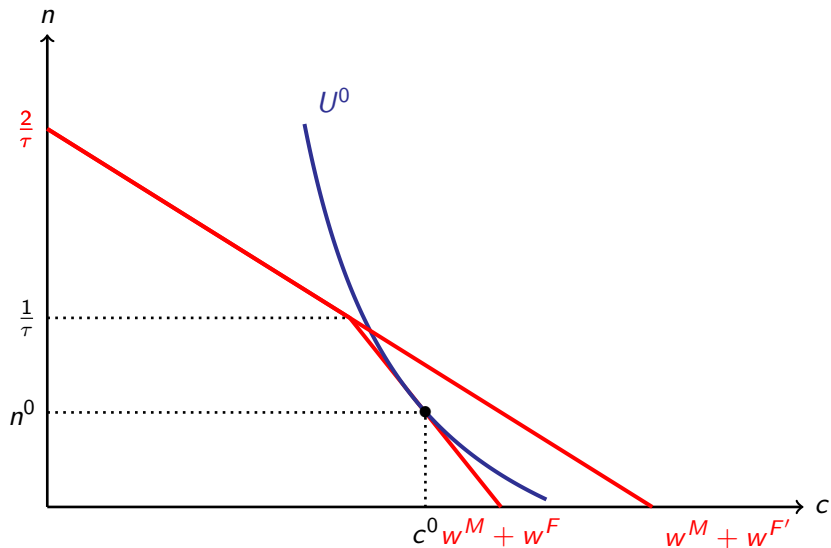
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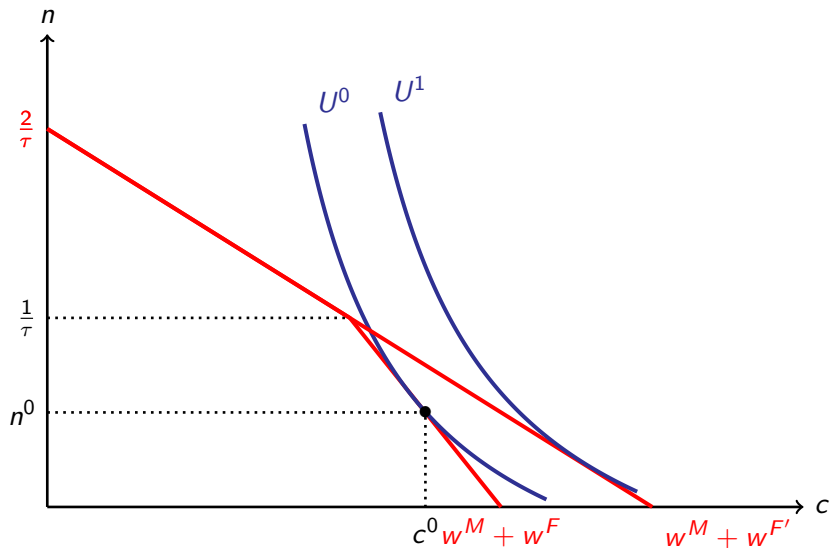
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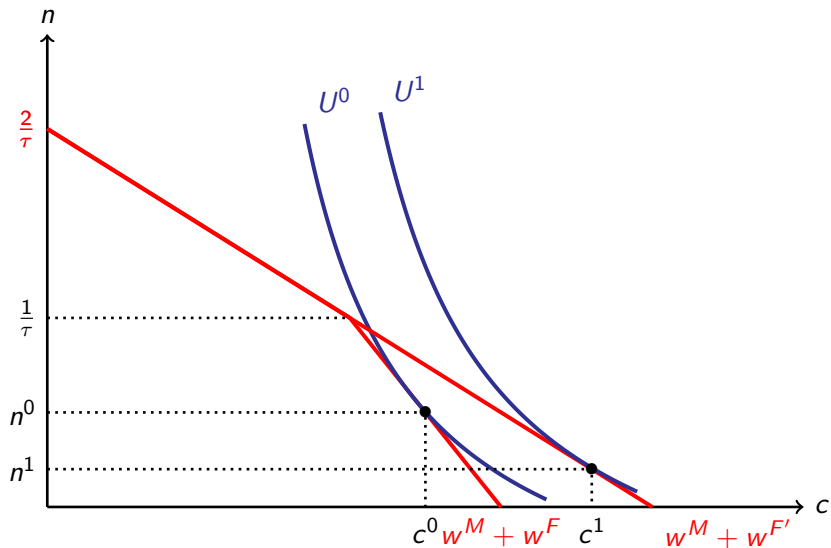
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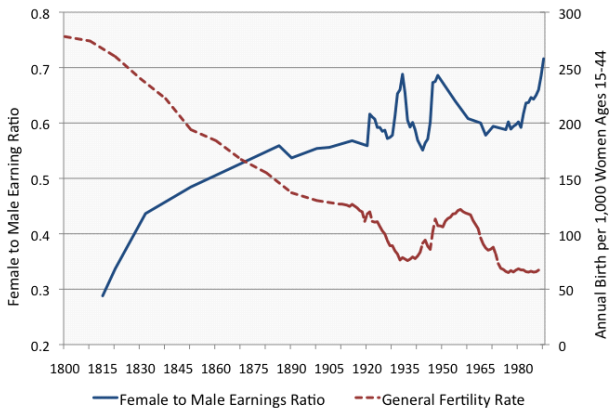
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Women's Relative Wages and Fertility - US



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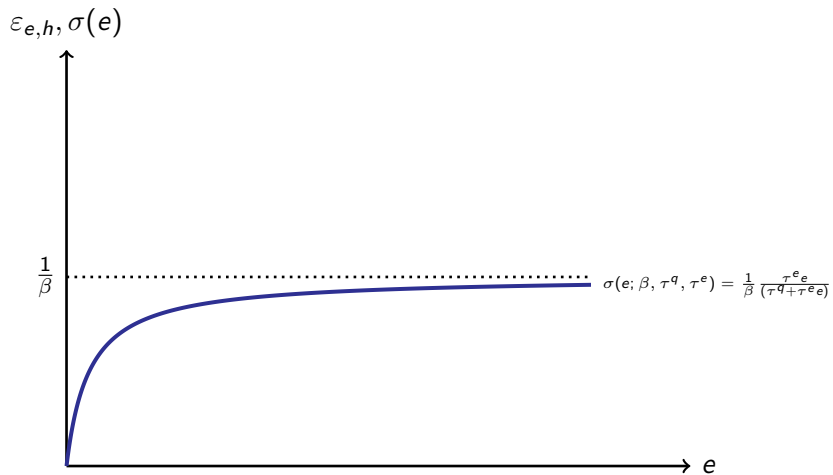
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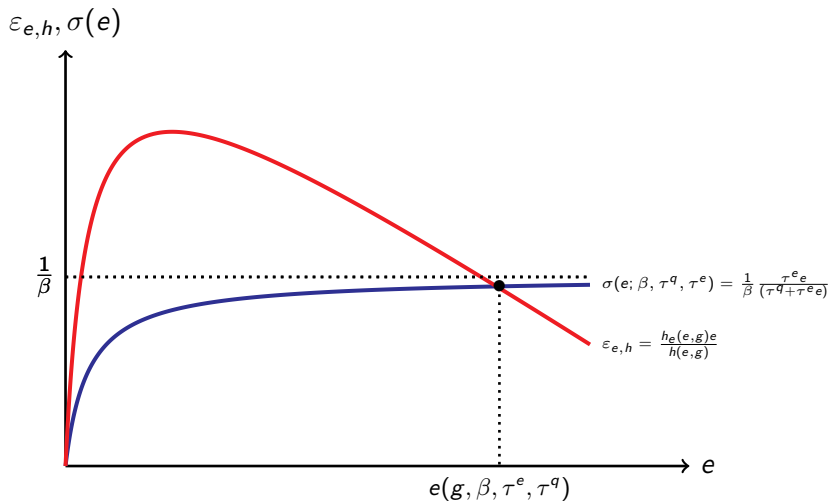
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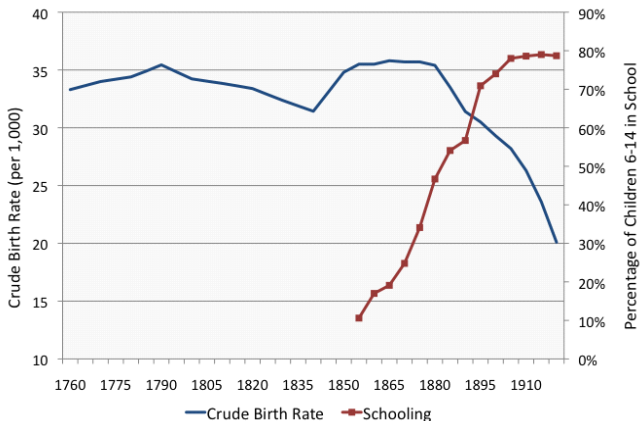
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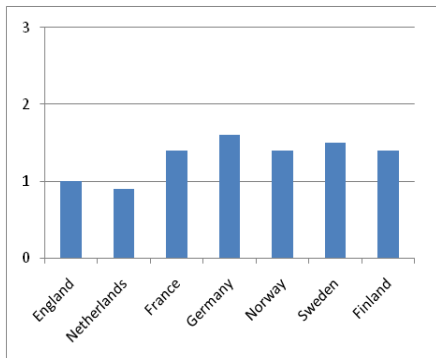
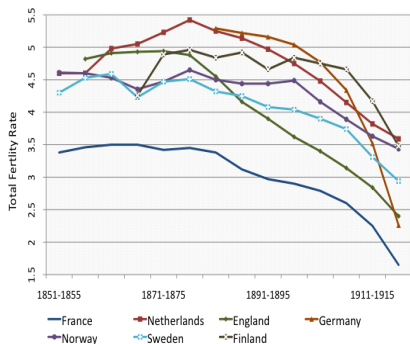
- The cost of educating a child increases and the elasticity of child quality with respect to the cost of child quality is smaller than one in absolute value

$$\partial n / \partial \tau^e < 0 \text{ if } [\partial e / \partial \tau^e][\tau^e / e] > -1$$

Human Capital Formation and the DT - England



Growth Rates 1870-1913 and DT



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- England (1580-1871) Adverse effect of family size on children's literacy. (Klemp-Weisdorf 2016)