The Demographic Transition

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Economic Growth and Comparative Development
Phases of Development: Standard of Living

- The Malthusian Epoch
Phases of Development: Standard of Living

- The Malthusian Epoch
- The Post-Malthusian Regime
Phases of Development: Standard of Living

- The Malthusian Epoch
- The Post-Malthusian Regime
- The Modern Growth Regime
Phases of Development: Timeline of the Most Developed Economies

Malthusian Epoch
(99.8%)

200K BP to 1750s
Phases of Development: Timeline of the Most Developed Economies

- **Malthusian Epoch** (99.8%)
  - 200K BP
  - 1750s

- **Post-Malthusian** (0.1%)
  - 1750s
  - 1870s
  - 2018
Phases of Development: Timeline of the Most Developed Economies

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- **Modern Growth** (0.1%)
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  - 2018
The positive relationship between income and population is reversed
The Demographic Transition

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- Fertility, mortality, and population growth decline very rapidly.
The Demographic Transition

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The Demographic Transition

1. The positive relationship between income and population is reversed.
2. Fertility, mortality and population growth decline very rapidly.
3. The impact of technological progress on output per capita are no longer counterbalanced by population growth.
4. Transition to Modern Growth.
Variation in years elapsed since the Onset of the Fertility Decline

The Demographic Transition

Definitions

The Demographic Transition

Growth & Comparative Development 7 / 72
The Demographic Transition – Definitions

- **Crude Birth Rate (CBR) or Birth Rate**
The Demographic Transition – Definitions

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\[ RNI = CBR - CDR \]
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  \[ RNI = CBR - CDR \]

  \[ \Rightarrow \text{population growth excluding migration} \]
Definition of Total Fertility Rate (TFR):

- **Total Fertility Rate (TFR)**
  - The average number of children that would be born to a woman over her lifetime if she experienced the exact current age-specific fertility rates through her lifetime and survived from birth to the end of her reproductive life (15-44/9).

Definition of Net Reproduction Rate (NRR):

- **Net Reproduction Rate (NRR)**
  - The number of daughters a woman would have in her lifetime if she were subject to prevailing age-specific fertility and mortality rates in the given year, experiencing the exact current age-specific fertility rates and the exact current age-specific mortality rates.
The Demographic Transition – Definitions

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Net reproduction rate (NRR)
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- **TFR & NRR are synthetic rates**
Early Fertility Decline – Western Offshoots
Early Fertility Decline – Western Europe
Early Fertility Decline – Eastern Europe
Late Fertility Decline – Latin America

The Demographic Transition

Evidence

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Growth & Comparative Development
Late Fertility Decline – Asia
Late Fertility Decline – Africa
The Demographic Transition in Western Europe: Total Fertility Rates
Mortality Decline Western Europe: 1730-1920
The Evolution of Total Fertility Rate across Regions, 1960-1999
Decline in infant mortality rates across regions, 1960-1999
Timing of the Demographic Transition and Current Income per Capita

Coef = 1.3462847, (robust) se = 0.10852591, t = 12.41
Theories of the Demographic Transition

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The Rise in Income

The cost of raising children is primarily parental time

The rise in income increased the opportunity cost of raising children

⇒ reduction in fertility

(Becker, 1960)

The income elasticity of child quality is larger than that of quantity

The rise in income

⇒ substitution of child quality for quantity

⇒ reduction in fertility

(Becker and Lewis, JPE 1973)

The Decline in Child Mortality

Decline in child mortality enabled families to attain their desirable number of children with lower number of births

The mortality decline

⇒ reduction in fertility
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Growth & Comparative Development
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- **The Old-Age Security Hypothesis** (Caldwell, 1976)
  - Children as a form of investment good (in the absence of access to financial markets)
  - Development of financial markets reduced the demand for children as an investment good
  - ⇒ reduction in fertility

- **The Decline in the Gender Wage Gap** (Galor-Weil, AER 1996)
  - The process of development decreased the gender wage gap
  - The rise in the relative wages of women increased the opportunity cost of raising children more than family income
  - ⇒ reduction in fertility
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Theories of the Demographic Transition

The Rise in Human Capital Formation:

(Galor-Weil, AER 2000)

The rise in the industrial demand for human capital induced human capital formation.

The presence of a budget constraint $\Rightarrow$ substitution of child quality for quantity $\Rightarrow$ reduction in fertility.
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    $\Rightarrow$ reduction in fertility
The Rise in Income - Main Hypothesis

- The cost of raising children is primarily parental time.
- The rise in income increased the opportunity cost of raising children, leading to a reduction in fertility (Becker, 1960).

- The income elasticity of child quality is larger than that of quantity.
- The rise in income leads to a substitution of child quality for quantity, resulting in a reduction in fertility (Becker and Lewis, JPE 1973).
The Rise in Income - Main Hypothesis

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$\Rightarrow$ substitution of child quality for quantity $\Rightarrow$ reduction in fertility

(Becker and Lewis, JPE 1973)
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  - The rise in income \(\Rightarrow\) substitution of child quality for quantity \(\Rightarrow\) reduction in fertility (Becker and Lewis, JPE 1973)
The Rise in Income: Income and Fertility (Again!)

- Child rearing is time-intensive
The Rise in Income: Income and Fertility (Again!)

- Child rearing is time-intensive
- Household’s Budget constraint

\[ y\tau n + c \leq y \]
The Rise in Income: Income and Fertility (Again!)

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- Household’s Budget constraint

\[ y \tau n + c \leq y \]

- \( y \equiv \) household’s income
The Rise in Income: Income and Fertility (Again!)

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\[ y\tau n + c \leq y \]

- \( y \equiv \) household’s income
- \( c \equiv \) household’s consumption
The Rise in Income: Income and Fertility (Again!)

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- \( c \equiv \) household’s consumption
- \( n \equiv \) household’s children
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- \( \tau \equiv \) time cost per child
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- \( y\tau \equiv \) opportunity cost of raising a child
The Rise in Income: Income and Fertility (Again!)

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$$y\tau n + c \leq y$$

- $y \equiv$ household’s income
- $c \equiv$ household’s consumption
- $n \equiv$ household’s children
- $\tau \equiv$ time cost per child
- $y\tau \equiv$ opportunity cost of raising a child

Equivalently

$$c \leq y(1 - \tau n)$$

- $1 \equiv$ household’s time endowment
The Rise in Income: Income and Fertility (Again!)

- Child rearing is time-intensive
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\[ y\tau n + c \leq y \]

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- Equivalently

\[ c \leq y(1 - \tau n) \]

- \( 1 \equiv \) household’s time endowment
- \( (1 - \tau n) \equiv \) labor force participation
The Rise in Income: Income and Fertility (Again!)

- Child rearing is time-intensive
- Household’s Budget constraint

\[ y_\tau n + c \leq y \]

- \( y \equiv \) household’s income
- \( c \equiv \) household’s consumption
- \( n \equiv \) household’s children
- \( \tau \equiv \) time cost per child
- \( y_\tau \equiv \) opportunity cost of raising a child

Equivalently

\[ c \leq y (1 - \tau n) \]

- \( 1 \equiv \) household’s time endowment
- \( (1 - \tau n) \equiv \) labor force participation
- \( \tau n \equiv \) time spent raising children
Rise in Income: Optimal Choice
Rise in Income: Optimal Choice

\[ n\tau \\
\]

\[ U^0 \]

\[ y^0 \]

\[ c \]
Rise in Income: Optimal Choice

\[ n \tau \]

\[ \tau n^0 \]

\[ c^0 \]

\[ y^0 \]

\[ U^0 \]
Rise in Income: Optimal Choice

\[ \begin{align*}
U_0 &= \tau n_0 c_0 y_0^0 \\
U_1 &= \tau n_1 c_1 y_1^1 \\
U_2 &= \tau n_2 c_2 y_2^2 \\
U_3 &= \tau n_3 c_3 y_3^3
\end{align*} \]
Rise in Income: Optimal Choice

\[ U_0 \cdot \tau n_0 \]

\[ U^1 \]

\[ c^0 \]

\[ y^0 \]

\[ y^1 \]
Rise in Income: Optimal Choice

\[ \begin{align*}
    n^\tau &= y^0 - c^0 \\
    n^\tau &= y^1 - c^1 \\
    n^\tau &= y^2 - c^2
\end{align*} \]
Rise in Income: Optimal Choice

\[ \begin{align*}
  U_0 & \cdot \tau_0 \cdot c_0 \\
  U_1 & \cdot \tau_1 \cdot c_1 \\
  U_2 & \cdot \tau_2 \cdot c_2 \\
  U_3 & \cdot \tau_3 \cdot c_3 \\
\end{align*} \]
Rise in Income: Optimal Choice

\[ \begin{align*}
\frac{\partial U_0}{\partial c} & = n \tau \\
\frac{\partial U_0}{\partial n} & = c_0 \\
\frac{\partial U_0}{\partial y} & = y_0
\end{align*} \]
Rise in Income: Optimal Choice

\[ \begin{align*}
U_0 & \cdot \tau^n  \\
U_1 & \cdot \tau^n  \\
U_2 & \cdot \tau^n  \\
U_3 & \cdot \tau^n
\end{align*} \]

\[ \begin{align*}
c_0 &  \\
c_1 &  \\
c_2 &  \\
y_0 &  \\
y_1 &  \\
y_2 & 
\end{align*} \]
Rise in Income: Optimal Choice

\[ \frac{\partial n}{\partial n} \]

\[ n \tau \]

\[ U^0 \]

\[ U^1 \]

\[ U^2 \]

\[ y_0 \]

\[ y^1 \]

\[ y^2 \]

\[ y^3 \]

\[ c^0 \]

\[ c^1 \]

\[ c^2 \]

\[ \tau \]
Rise in Income: Optimal Choice

\[ n\tau \]

\[ \frac{\tau n^2}{n^2} \]

\[ c_0 \quad c_1 \quad c_2 \]

\[ y_0 \quad y_1 \quad y_2 \quad y_3 \]
Rise in Income: Optimal Choice

\[ n \tau \]

\[ \begin{align*}
\tau_1 y_0 & \Rightarrow U_0 \\
\tau_2 y_1 & \Rightarrow U_1 \\
\tau_3 y_2 & \Rightarrow U_2 \\
\tau_4 y_3 & \Rightarrow U_3
\end{align*} \]
The Rise in Income: Mechanism

- The rise in income generates two conflicting effects:
The Rise in Income: Mechanism

- The rise in income generates two conflicting effects:
  - An income effect:
    - More income can be devoted to raising children operates towards $n \uparrow$
    - The opportunity cost of raising children increases operates towards $n \downarrow$

\[ y + c \leq \left[ y \right] \]

$\text{More income can be devoted to raising children}$

$\text{The opportunity cost of raising children increases}$
The rise in income generates two conflicting effects:

- An income effect:
  \[ y \tau n + c \leq [y] \uparrow \]

- A substitution effect:
  \[ y^{\tau n} \leq [y] \downarrow \]
The Rise in Income: Mechanism

- The rise in income generates two conflicting effects:
  - An income effect:
    \[ y\tau n + c \leq y \uparrow \]
    
    More income can be devoted to raising children
The rise in income generates two conflicting effects:

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  \[ y \tau n + c \leq [y] \uparrow \]
  
  - More income can be devoted to raising children
  - operates towards \( n \uparrow \)
The rise in income generates two conflicting effects:

- An income effect:
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  operates towards \( n \uparrow \)

- A substitution effect:
The rise in income generates two conflicting effects:

- An income effect:
  \[ y\tau n + c \leq [y] \uparrow \]
  - More income can be devoted to raising children
  - operates towards \( n \uparrow \)

- A substitution effect:
  \[ \uparrow [y\tau]n + c \leq y \]
The rise in income generates two conflicting effects:

- An income effect:
  \[ y\tau n + c \leq [y] \uparrow \]
  - More income can be devoted to raising children
  - Operates towards \( n \uparrow \)

- A substitution effect:
  \[ \uparrow [y\tau]n + c \leq y \]
  - The opportunity cost of raising children increases
The rise in income generates two conflicting effects:

- An income effect:
  
  \[ y\tau n + c \leq [y] \uparrow \]

  - More income can be devoted to raising children
  - Operates towards \( n \uparrow \)

- A substitution effect:
  
  \[ \uparrow [y\tau]n + c \leq y \]

  - The opportunity cost of raising children increases
  - Operates towards \( n \downarrow \)
The Rise in Income: Mechanism

- The substitution effect dominates at a higher level of income
The Rise in Income: Mechanism

- The substitution effect dominates at a higher level of income.
- As income increases fertility declines.
The Rise in Income: Mechanism

- The substitution effect dominates at a higher level of income
- As income increases fertility declines
- Fertility declines in the process of development
The Rise in Income - Theoretical Evaluation

- Preference-based theory (unattractive)
The Rise in Income - Theoretical Evaluation

- Preference-based theory (unattractive)
  - Innate bias against child quantity beyond a certain level of income - non-refutable
The Rise in Income - Theoretical Evaluation

- Preference-based theory (unattractive)
  - Innate bias against child quantity beyond a certain level of income - non-refutable
  - Non-robust (e.g., the class of homothetic preferences will not trigger a fertility decline)
The Rise in Income - Homothetic Preferences

- Preferences:

\[ u = n^\gamma c^{(1-\gamma)} \quad \text{with} \quad 0 < \gamma < 1 \]
The Rise in Income - Homothetic Preferences

- Preferences:
  \[ u = n^\gamma c^{(1-\gamma)} \quad 0 < \gamma < 1 \]

- Budget constraint
  \[ y\tau n + c \leq y \]
The Rise in Income - Homothetic Preferences

Preferences:
\[ u = n^\gamma c^{(1-\gamma)} \quad 0 < \gamma < 1 \]

Budget constraint
\[ y\tau n + c \leq y \]

Optimization: (fraction \( \gamma \) of income is spent on children and \( (1 - \gamma) \) on consumption)
The Rise in Income - Homothetic Preferences

Preferences:
\[ u = n^\gamma c^{(1-\gamma)} \quad \text{0} < \gamma < 1 \]

Budget constraint
\[ y\tau n + c \leq y \]

Optimization: (fraction $\gamma$ of income is spent on children and $(1 - \gamma)$ on consumption)
\[ y\tau n = \gamma y \]
\[ c = (1 - \gamma)y \]
The Rise in Income - Homothetic Preferences

Optimal number of children

\[ n = \frac{\gamma}{\tau} \]
The Rise in Income - Homothetic Preferences

- Optimal number of children
  \[ n = \frac{\gamma}{\tau} \]

- The rise in income has no effect on fertility, i.e.,
  \[ |\text{Income effect}| = |\text{Substitution effect}| \]
The Rise in Income - Homothetic Preferences

- Optimal number of children
  \[ n = \frac{\gamma}{\tau} \]

- The rise in income has no effect on fertility, i.e.,
  \[ |\text{Income effect}| = |\text{Substitution effect}| \]

- Fertility is unaffected by the process of development
The Rise in Income - Homothetic Preferences

The graph shows a straight line that passes through the points (0, 1) and (1, 0) on the axes labeled $n\tau$ and $c$, respectively. The line's equation can be represented as $c = \gamma y$, where $\gamma$ is a constant.
The Rise in Income - Homothetic Preferences

\[
U^0 \cdot \tau^0 = c^0 \gamma^0
\]

\[
U^1 \cdot \tau^1 = c^1 \gamma^1
\]

\[
U^2 \cdot \tau^2 = c^2 \gamma^2
\]

\[
U^3 \cdot \tau^3 = c^3 \gamma^3
\]
The Rise in Income - Homothetic Preferences

\[ U_0 = \int_0^{\tau_0} (c/n)^{\gamma} \, dc \]

\[ U_1 = \int_0^{\tau_1} (c/n)^{\gamma_1} \, dc \]

\[ U_2 = \int_0^{\tau_2} (c/n)^{\gamma_2} \, dc \]

\[ U_3 = \int_0^{\tau_3} (c/n)^{\gamma_3} \, dc \]
The Rise in Income - Homothetic Preferences
The Rise in Income - Homothetic Preferences

\[ U_0^n = c^n \gamma \]

\[ U_1^n = c^n \gamma \]

\[ U_2^n = c^n \gamma \]

\[ U_3^n = c^n \gamma \]

\[ c^n = c^n \gamma \]

\[ y^n = y^n \gamma \]

\[ y^n = y^n \gamma \]

\[ \tau n^0 \]

\[ n^\tau \]

\[ c^0 \]

\[ y^0 \]

\[ y^1 \]
The Rise in Income - Homothetic Preferences

$$n^\tau_c \cdot n^\tau_{c_0} \cdot n^\tau_{c_1} \cdot n^\tau_{c_2}$$
The Rise in Income - Homothetic Preferences

\[ u_0 (c^n_0, y^n_0) = c^n_0 \]

\[ u_1 (c^n_1, y^n_1) = c^n_1 \]

\[ u_2 (c^n_2, y^n_2) = c^n_2 \]

\[ u_3 (c^n_3, y^n_3) = c^n_3 \]

\( \tau n^1 = \tau n^0 \)

\( \tau n^1 = \tau n^0 \)
The Rise in Income - Homothetic Preferences

\[
\tau n^1 = \tau n^0
\]

\[
c^0 \quad c^1 \quad y^0 \quad y^1 \quad y^2
\]
The Rise in Income - Homothetic Preferences

\[ \tau n^2 = \tau n^1 = \tau n^0 \]

\[ U^0 c^0 + \left( \frac{1}{\alpha} \right) y^0 = U^1 c^1 + \left( \frac{1}{\alpha} \right) y^1 = U^2 c^2 + \left( \frac{1}{\alpha} \right) y^2 \]
The Rise in Income - Homothetic Preferences

\[ \tau n^2 = \tau n^1 = \tau n^0 \]

\[ n^\tau \]

\[ c^0 \quad c^1 \quad c^2 \quad y^0 \quad y^1 \quad y^2 \]
The Rise in Income - Homothetic Preferences

\[ \begin{align*}
U_0^c &= \tau_0 n^0 \\
U_1^c &= \tau_1 n^1 \\
U_2^c &= \tau_2 n^2 \\
U_3^c &= \tau_3 n^3
\end{align*} \]
The Rise in Income - Homothetic Preferences

\[ U^n = c^n \]
The Rise in Income - Homothetic Preferences

\[ \tau n^3 = \tau n^2 = \tau n^1 = \tau n^0 \]

\[ U^3 \]

\[ U^2 \]

\[ U^1 \]

\[ U^0 \]
The Rise in Income: Testable predictions

Across countries that are similar in sociocultural characteristics (and thus in noneconomic factors that may affect fertility decisions), the timing of the fertility decline is inversely related to the level of income per capita.
Across countries that are similar in sociocultural characteristics (and thus in noneconomic factors that may affect fertility decisions), the timing of the fertility decline is inversely related to the level of income per capita.

Within an economy, the number of (surviving) children across households is inversely related to their levels of income.
Cross Section of Countries (1870-2000) - Income per worker was positively associated with fertility rates, accounting for mortality rates and education (Murtin 2013).
The Rise in Income: Refuting Cross Country Evidence

- Cross Section of Countries (1870-2000) - Income per worker was positively associated with fertility rates, accounting for mortality rates and education (Murtin 2013).

- Western Europe (1870s) The DT occurred among countries that differed significantly in their income per capita.
Simultaneous DT across European Countries that Differ in Income per Capita
The Rise in Income: Refuting Evidence from Individual Countries

- France (1876–96): Income per capita had a positive effect on fertility rates during France’s demographic transition, accounting for education, the gender literacy gap, and mortality rates (Murphy 2015).
The Rise in Income: Refuting Evidence from Individual Countries

- **France (1876–96)** Income per capita had a positive effect on fertility rates during France’s demographic transition, accounting for education, the gender literacy gap, and mortality rates (Murphy 2015)

- **England (During the DT):** The force associated with the rise in income would have led to an increase in fertility rates (Fernandez-Villaverde 2001)
The Rise in Income: Refuting Evidence from Individual Countries

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- England (During the DT): The force associated with the rise in income would have led to an increase in fertility rates (Fernandez-Villaverde 2001)

- England (1630s) Reproductive success increases with income (Clark and Hamilton JEH 2006)
The Decline in Child Mortality - Main Hypothesis

Parents generate utility from the number of surviving children. A decline in child mortality permits parents to reach a given level of surviving children with lower fertility. The decline in mortality triggered the subsequent decline in fertility.
The Decline in Child Mortality - Main Hypothesis

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- A decline in child mortality permits parents to reach a given level of surviving children with lower fertility
- The decline in mortality triggered the subsequent decline in fertility
The Decline in Mortality – Mechanism

Theories

The Decline in Child Mortality

\[ u = n \gamma c (1 - \gamma) \]

\[ c \equiv \text{household's consumption} \]

\[ n \equiv \text{household's surviving children} \]

Survival children

\[ n = \theta n_b \]

\[ \theta \equiv \text{probability of a child to survive infancy} \]

\[ n_b \equiv \text{household's children born} \]
The Decline in Mortality – Mechanism

Preferences:

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The Decline in Mortality – Mechanism

\[ y \tau + c \leq y \]

\( y \equiv \) household's income

\( c \equiv \) household's consumption

\( \tau \equiv \) time cost of raising a surviving child

\( y \tau \equiv \) opportunity cost of raising a surviving child
The Decline in Mortality – Mechanism

• Budget constraint

\[ yτn + c \leq y \]
The Decline in Mortality – Mechanism

- Budget constraint
  \[ y\tau n + c \leq y \]

- \( y \equiv \text{household’s income} \)
The Decline in Mortality – Mechanism

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- \( c \equiv \) household’s consumption
The Decline in Mortality – Mechanism

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- Optimization: (fraction $\gamma$ of income is spent on children and $(1 - \gamma)$ on consumption)
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\[
y\tau n = \gamma y \\
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The Decline in Mortality – Mechanism

- Optimization: (fraction $\gamma$ of income is spent on children and $(1 - \gamma)$ on consumption)
  
  $$y \tau n = \gamma y$$
  
  $$c = (1 - \gamma)y$$

- Optimal number of surviving children (NRR)
  
  $$n = \gamma / \tau$$
The Decline in Mortality – Mechanism

- Optimization: (fraction $\gamma$ of income is spent on children and $(1 - \gamma)$ on consumption)
  \[ y\tau n = \gamma y \]
  \[ c = (1 - \gamma)y \]

- Optimal number of surviving children (NRR)
  \[ n = \frac{\gamma}{\tau} \]

- Optimal fertility (# of successful pregnancies - TFR)
  \[ n^b = \frac{n}{\theta} = \frac{\gamma}{(\tau \theta)} \]
The Decline in Mortality – Testable Predictions

- Child mortality rate, \((1 - \theta)\), has a positive effect on TFR
The Decline in Mortality – Testable Predictions

- Child mortality rate, $(1 - \theta)$, has a positive effect on TFR
- Child mortality rate, $(1 - \theta)$, has no effect on NRR
The Decline in Child Mortality

- Worldwide: NRR and TFR plummet jointly during the demographic transition (Lehr 2009). But the theory does not predict a decline in NRR
The Decline in Child Mortality

- **Worldwide:** NRR and TFR plummet jointly during the demographic transition (Lehr 2009). But the theory does not predict a decline in NRR.

- **NRR does not decline unless:**

  - There exists a precautionary demand for children RA with respect to fertility
  - \[ RA_{\text{with respect to consumption}} \]
  - Replacement fertility is insignificant (empirical estimates 0.2–0.6)
  - Resources saved from investment in non-surviving children are not channeled towards higher fertility
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  - RA with respect to fertility > RA with respect to consumption
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The Decline in Mortality and Fertility - Evidence

The Decline in Child Mortality

The Demographic Transition

Growth & Comparative Development 43 / 72
The Decline in Child Mortality – Challenges to the Theory

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- **Western Europe:** No change in the patterns of mortality decline at the time of the sharp decline in fertility

- **England:** The decline in mortality started in the 1730s (140 years before the fertility decline) and was accompanied by a steady increase in fertility rates until 1800
The Decline in Mortality: Refuting Evidence from Individual Countries

- **France** (1876–96): Mortality rate had no effect on fertility during France’s demographic transition, accounting for education, income, and the gender literacy gap. (Murphy 2009)
The Decline in Mortality: Refuting Evidence from Individual Countries

- **France (1876–96):** Mortality rate had no effect on fertility during France’s demographic transition, accounting for education, income, and the gender literacy gap. (Murphy 2009)

- **England (1861–1951):** The force associated with the decline in child mortality would have led to an increase in fertility rates (Fernandez-Villaverde 2001; Doepke 2005)
The Old-Age Security Hypothesis

Children is a form of investment good (in the absence of capital markets). The development of financial markets reduced the demand for children for investment purposes and triggered a decline in fertility.
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The Old-Age Security Hypothesis - Challenges to the Theory

- The decline in the importance of old-age support is unlikely to be a major force behind the significant reduction in fertility – at a rate of 30–50% – during the demographic transition:
The Old-Age Security Hypothesis - Challenges to the Theory

- The decline in the importance of old-age support is unlikely to be a major force behind the significant reduction in fertility – at a rate of 30–50% – during the demographic transition:
  - Rare examples in nature of offspring that support their parents in old age

- Institutions supporting individuals in their old age were formed well before the demographic transition
- England (16th century) Parents did not rely on support from children in their old age
  - Richer individuals who presumably had better access to financial markets, had a larger number of surviving children
The Old-Age Security Hypothesis - Challenges to the Theory

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- Institutions supporting individuals in their old age were formed well before the demographic transition
  - England (16th century) Parents did not rely on support from children in their old age (Pelling and Smith 1991)
The Old-Age Security Hypothesis - Challenges to the Theory

- The decline in the importance of old-age support is unlikely to be a major force behind the significant reduction in fertility – at a rate of 30–50% – during the demographic transition:
  - Rare examples in nature of offspring that support their parents in old age

- Institutions supporting individuals in their old age were formed well before the demographic transition
  - England (16th century) Parents did not rely on support from children in their old age (Pelling and Smith 1991)

- Prior to the demographic transition, richer individuals who presumably had better access to financial markets, had larger number of surviving children
The Decline in the Gender Wage Gap

The inevitable rise in the relative wages of women in the process of development increases the opportunity cost of raising children more than family income ⇒ reduction in fertility.

Ömer Özak

The Demographic Transition

Growth & Comparative Development
The Decline in the Gender Wage Gap

- The inevitable rise in the relative wages of women in the process of development
The Decline in the Gender Wage Gap

- The inevitable rise in the relative wages of women in the process of development
  - increases the opportunity cost of raising children more than family income
The inevitable rise in the relative wages of women in the process of development

- increases the opportunity cost of raising children more than family income

⇒ reduction in fertility
Mechanism: I. Development and Women’s Wages

Female-Biased Technical change
Mechanization and advanced technologies have complemented mental tasks more than physical tasks.
Women have physiological comparative advantage in mental (rather than physical) tasks.

\[ \text{The process of development has (inevitably) increased the productivity of } \]
\[ \text{women relative to men: } \]
\[ \text{Economic Development } \Rightarrow \]
\[ \left( \frac{w_F}{w_M} \right) \uparrow \]

\[ w_F \equiv \text{women’s wages} \]
\[ w_M \equiv \text{men’s wages} \]
Mechanism: I. Development and Women’s Wages

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\[
\text{Economic Development} \implies \left( \frac{w^F}{w^M} \right) \uparrow
\]
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Mechanism: I. Development and Women’s Wages

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- \( w^M \equiv \) men’s wages
Evolution of the Gender Earnings Ratio - US

Theories
The Decline in the Gender Wage Gap
Evolution of the Gender Literacy Gap - England

The Literacy Rates for males and females in England from 1840 to 1900.

- Male literacy rates are shown in blue.
- Female literacy rates are shown in red.

The chart indicates a gradual increase in literacy rates for both genders over the years, with females catching up with males towards the end of the period.
Mechanism: Women’s Relative Wages and Fertility

Child rearing is time-intensive. Women are the prime care-takers engaged in child rearing. Budget constraint (if only women raise children):

\[ w_F \tau_n + c \leq w_M + w_F \]

where:
- \( w \) ≡ household’s income
- \( c \) ≡ household’s consumption
- \( n \) ≡ household’s (surviving) children
- \( \tau \) ≡ time cost per child
- \( w_F \tau \) ≡ opportunity cost of raising a child
Mechanism: Women’s Relative Wages and Fertility

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\[ w^F \tau n + c \leq w^M + w^F \]

- \( w^F + w^M \equiv \text{household’s income} \)
Mechanism: Women’s Relative Wages and Fertility

- Child rearing is time-intensive
- Women are the prime care-takers engaged in child rearing
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\[ w^F \tau n + c \leq w^M + w^F \]

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- \( c \equiv \) household’s consumption
- \( n \equiv \) household’s (surviving) children
- \( \tau \equiv \) time cost per child
- \( w^F \tau \equiv \) opportunity cost of raising a child
Mechanism: Women’s Relative Wages and Fertility

\[ \frac{2}{\tau} \]

\[ \frac{1}{\tau} \]

\[ n \]

\[ c \]

\[ w^M + w^F \]
Mechanism: Women’s Relative Wages and Fertility
Mechanism: Women’s Relative Wages and Fertility

\[ \frac{2}{\tau} \]

\[ \frac{1}{\tau} \]

\[ n^0 \]

\[ U^0 \]

\[ c^0 w^M + w^F \]
Mechanism: Women’s Relative Wages and Fertility
Mechanism: Women’s Relative Wages and Fertility

\[ n \]

\[ \frac{2}{\tau} \]

\[ \frac{1}{\tau} \]

\[ n^0 \]

\[ U^0 \]

\[ U^1 \]

\[ c^0 w^M + w^F \]

\[ w^M + w^{F'} \]
Mechanism: Women's Relative Wages and Fertility

\[ c_0 \tau w^M + w^F \]

\[ n^0 \]

\[ n^1 \]

\[ c^1 \]

\[ w^M + w^{F'} \]
Mechanism: Women’s Relative Wages and Fertility
Mechanism: Women’s Relative Wages and Fertility

- The rise in women’s wages, $w^F$, generates two conflicting effects:
Mechanism: Women’s Relative Wages and Fertility

- The rise in women’s wages, $w^F$, generates two conflicting effects:
  - An income effect:
    \[ w^F \tau n + c \leq w^M + [w^F] \uparrow \]
Mechanism: Women’s Relative Wages and Fertility

- The rise in women’s wages, $w^F$, generates two conflicting effects:
  - An income effect:
    \[
    w^F \tau n + c \leq w^M + [w^F] \uparrow
    \]
    - More income for raising children

- A rise in men’s wages generate only an income effect
  \[
  w^F \tau n + c \leq w^M + [w^F] \uparrow
  \]
Mechanism: Women’s Relative Wages and Fertility

- The rise in women’s wages, $w^F$, generates two conflicting effects:
  - An income effect:
    $$w^F \tau n + c \leq w^M + [w^F] \uparrow$$
  - More income for raising children $\Rightarrow$ operates towards $n \uparrow$
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The Decline in the Gender Wage Gap

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Women’s Relative Wages and Fertility - US

The diagram shows a comparison between the Female to Male Earnings Ratio and the General Fertility Rate over time in the US. The X-axis represents years from 1800 to 1980, while the Y-axis represents the Female to Male Earnings Ratio (0.2 to 0.8) and the Annual Birth per 1,000 Women Ages 15-44 (0 to 300). The graph indicates a decline in the wage gap and a corresponding increase in fertility rates over the centuries.
Women’s Relative Wages and Fertility - Evidence

- US (1970s): $w^F \uparrow \implies n \downarrow$ & $w^M \uparrow \implies n \uparrow$ (Heckman and Walker ECT 79)

- Sweden’s demographic transition: $(w^F/w^M) \uparrow \implies n \downarrow$ (Schultz 1985)

- France (1876–1896): reduction in the gender literacy gap had an adverse effect on fertility, accounting for income per capita, educational attainment, and mortality rates (Murphy 2015)
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The Rise in the Demand for Human Capital - Main Thesis

The acceleration in the rate of technological progress in the 2nd phase of industrialization increased the demand for human capital. Education enabled individuals to cope with a rapidly changing technological environment. The rise in the demand for human capital induced a substitution of quality for quantity of children, triggering a demographic transition: reduction in fertility.
The Rise in the Demand for Human Capital - Main Thesis

- The acceleration in the rate of technological progress in the 2nd phase of industrialization increased the demand for human capital.
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The rise in the demand for human capital induced a substitution of quality for quantity of children triggering a demographic transition:

⇒ reduction in fertility
The Model - Preferences

\[ u = (1 - \gamma) \ln c + \gamma [\ln n + \beta \ln h] \]
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The Model - Preferences

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- \( n \equiv \) (surviving) children
- \( h \equiv \) quality (human capital) of each child
- \( \beta \equiv \) degree of preference for child quality; \( \beta < 1 \)
The Model - Budget Constraint

\[ yn(\tau^q + \tau^e e) + c \leq y \]
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- \( y \equiv \) household potential income
The Model - Budget Constraint

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\begin{itemize}
  \item \( y \equiv \) household potential income
  \item \( \tau^q \equiv \) fraction of the household’s unit-time endowment required to raise a child, regardless of quality
\end{itemize}
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- \( (\tau^q + \tau^e e) \equiv \) time cost of raising a child with a level of education (quality) \( e \)
- \( y(\tau^q + \tau^e e) \equiv \) opportunity cost of raising a child with quality \( e \)
The Model - Human Capital Formation

\[ h = h(e, g) \]
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- \( h_e(e, g) > 0 \) & \( h_{ee}(e, g) < 0 \)
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- \( h_g(e, g) < 0 \) & \( h_{gg}(e, g) > 0 \)
The Rise in Human Capital Formation

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The Theories of Human Capital Formation

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- \( h_{eg}(e, g) > 0 \)
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- \( h(0, g) > 0 \) & \( \lim_{e \to 0} h_e(e, g) = \infty \); \( \lim_{e \to \infty} h_e(e, g) = 0 \)

Basic level of human capital & interior solution

\( \ddot{O} \text{meer} \ddot{O} \text{zak} \)

The Demographic Transition
Growth & Comparative Development 61 / 72
The Model - Human Capital Formation

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The Model - Optimization

\begin{equation}
\{n, e, c\} = \arg \max \gamma \left[ \ln n + \beta \ln h(e, g) \right] + (1 - \gamma) \ln c \quad \text{s.t.} \quad c \leq y \iff \{n, e\} = \arg \max \gamma \left[ \ln n + \beta \ln h(e, g) \right] + (1 - \gamma) \ln y \quad \text{s.t.} \quad c \leq y.
\end{equation}
The Model - Optimization

\[ \{ n, e, c \} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln c \]

s.t. \[ yn(\tau^q + \tau^e e) + c \leq y \]
The Model - Optimization

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\{n, e, c\} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln c
\]
\[s.t. \quad yn(\tau^q + \tau^e e) + c \leq y\]

since \(c = y[1 - n(\tau^q + \tau^e e)]\)
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Optimization

\[
\{ n, e \} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y [1 - n(\tau^q + \tau^e e)]
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Optimization

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\{n, e\} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y[1 - n(\tau^q + \tau^e e)]
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- with respect to \(n\):
Optimization

\[ \{ n, e \} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y[1 - n(\tau^q + \tau^e e)] \]

- with respect to \( n \):

\[ \gamma n = (1 - \gamma)y(\tau^q + \tau^e e) \]

\[ \frac{\gamma n}{n} = \frac{(1 - \gamma)y(\tau^q + \tau^e e)}{y[1 - n(\tau^q + \tau^e e)]} \]
\{ n, e \} = \text{arg max } \gamma[\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y[1 - n(\tau^q + \tau^e e)]

with respect to \( n \):

\[
\frac{\gamma}{n} = \frac{(1 - \gamma)y(\tau^q + \tau^e e)}{y[1 - n(\tau^q + \tau^e e)]}
\]

\[
\gamma[1 - n(\tau^q + \tau^e e)] = (1 - \gamma)(\tau^q + \tau^e e)n
\]
Optimization

\[ \{ n, e \} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y[1 - n(\tau^q + \tau^e e)] \]

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\[ \frac{\gamma}{n} = \frac{(1 - \gamma)y(\tau^q + \tau^e e)}{y[1 - n(\tau^q + \tau^e e)]} \]

\[ \gamma[1 - n(\tau^q + \tau^e e)] = (1 - \gamma)(\tau^q + \tau^e e)n \]

\[ n(\tau^q + \tau^e e) = \gamma \]
Optimization

\[ \{ n, e \} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y[1 - n(\tau^q + \tau^e e)] \]
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\{ n, e \} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y[1 - n(\tau^q + \tau^e e)]
\]

with respect to \( e \):

\[
\frac{\gamma \beta h_e(e, g)}{h(e, g)} = \frac{(1 - \gamma)yn\tau^e}{y[1 - n(\tau^q + \tau^e e)]}
\]
Optimization

\[ \{ n, e \} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y [1 - n(\tau^q + \tau^e e)] \]

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- since \( n(\tau^q + \tau^e e) = \gamma \)
Theories  The Rise in Human Capital Formation

Optimization

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Theories

The Rise in Human Capital Formation

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\[ \frac{\gamma \beta h_e(e, g)}{h(e, g)} = n\tau^e \quad \implies \quad \frac{\beta h_e(e, g)}{h(e, g)} = \frac{\tau^e}{(\tau^q + \tau^e e)} \]
Theories

The Rise in Human Capital Formation

Optimization

\[\{n, e\} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y[1 - n(\tau^q + \tau^e e)]\]

- with respect to \(e\):
  \[\frac{\gamma \beta h_e(e, g)}{h(e, g)} = \frac{(1 - \gamma)yn\tau^e}{y[1 - n(\tau^q + \tau^e e)]}\]

- since \(n(\tau^q + \tau^e e) = \gamma\)
  \[\frac{\gamma \beta h_e(e, g)}{h(e, g)} = n\tau^e \quad \Rightarrow \quad \frac{\beta h_e(e, g)}{h(e, g)} = \frac{\tau^e}{(\tau^q + \tau^e e)}\]

\[\Rightarrow \varepsilon_{h,e} \equiv \frac{h_e(e, g)e}{h(e, g)} = \frac{1}{\beta} \frac{\tau^e e}{(\tau^q + \tau^e e)} \equiv \sigma(e; \beta, \tau^q, \tau^e)\]
Optimal education choice

\[ \varepsilon_{e,h,\sigma(e)} \]
Optimal education choice

\[ \varepsilon_{e, h}, \sigma(e) \]

\[ \sigma(e; \beta, \tau^q, \tau^e) = \frac{1}{\beta} \frac{\tau^e e}{(\tau^q + \tau^e e)} \]
**Optimal education choice**

\[ \sigma(e; \beta, \tau^q, \tau^e) = \frac{1}{\beta} \frac{\tau^e}{(\tau^q + \tau^e)} \]

\[ \varepsilon_{e,h} = \frac{h_e(e,g)e}{h(e,g)} \]
Optimization

So,
So,

\[ n = \gamma / (\tau^q + \tau^e e) \]

\[ \tau^e h(e, g) = \beta h_e(e, g)(\tau^q + \tau^e e) \]
Optimization

So,

\[ n = \frac{\gamma}{(\tau^q + \tau^e e)} \]

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Optimization

So,

\[ n = \gamma / (\tau^q + \tau^e e) \]

\[ \tau^e h(e, g) = \beta h_e(e, g)(\tau^q + \tau^e e) \]

\[ e = e(g, \beta, \tau^e, \tau^q) \]

\[ n = \gamma / [\tau^q + \tau^e e(g, \beta, \tau^e, \tau^q)] \]
Testable Predictions - Investment in Quality

The optimal level of investment in child quality increases if:

1. The technological environment changes more rapidly: \( \frac{\partial e(g,\beta,\tau_e,\tau_q)}{\partial g} > 0 \)
2. Preferences for child quality are higher: \( \frac{\partial e(g,\beta,\tau_e,\tau_q)}{\partial \beta} > 0 \)
3. The cost of raising a child (regardless of quality) increases: \( \frac{\partial e(g,\beta,\tau_e,\tau_q)}{\partial \tau_q} > 0 \)
4. The cost of educating a child decreases: \( \frac{\partial e(g,\beta,\tau_e,\tau_q)}{\partial \tau_e} < 0 \)
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Testable Predictions - Investment in Quality

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  \[
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  \]
- The cost of educating a child decreases
  \[
  \frac{\partial e(g, \beta, \tau^e, \tau^q)}{\partial \tau^e} < 0
  \]
Testable Predictions - Investment in Quantity

The optimal number of children decreases if:

- The technological environment changes more rapidly: \( \frac{\partial n}{\partial g} < 0 \)
- Preferences for child quality are higher: \( \frac{\partial n}{\partial \beta} < 0 \)
- The cost of raising a child (regardless of quality) increases: \( \frac{\partial n}{\partial \tau q} < 0 \)
- The cost of educating a child increases and the elasticity of child quality with respect to the cost of child quality is smaller than one in absolute value: \( \frac{\partial n}{\partial \tau e} < 0 \) if \( \left[ \frac{\partial e}{\partial \tau e} \right] \frac{\tau e}{e} > -1 \)
The optimal number of children decreases if:

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  \[ \frac{\partial n}{\partial \tau} q < 0 \]
- The cost of educating a child increases and the elasticity of child quality with respect to the cost of child quality is smaller than one in absolute value
  \[ \frac{\partial n}{\partial \tau e} < 0 \text{ if } \left( \frac{\partial e}{\partial \tau e} \right) \frac{\tau e}{e} > -1 \]
Testable Predictions - Investment in Quantity

The optimal number of children decreases if:

- The technological environment changes more rapidly
  \[ \frac{\partial n}{\partial g} < 0 \]
- Preferences for child quality are higher
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Theories  The Rise in Human Capital Formation

Human Capital Formation and the DT - England

![Graph showing Crude Birth Rate and Percentage of Children 6-14 in School over time (1760-1910).]
Growth Rates 1870-1913 and DT
Supporting Evidence: Cross-Country Evidence

- Cross Section of Countries (1870-2000) - educational attainment has been negatively associated with fertility, accounting for income per worker and mortality rates (Murtin 2013).
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- Cross Section of Countries (1960-1999): adverse effect on net fertility of an increase in productivity in advanced stages of development, when education demand dominates (Lehr 2009).
Supporting Evidence: Evidence from Individual Countries

- US (1910s): Eradication of hookworm – a positive shock to the return to child quality - had an adverse effect on fertility (Bleakley-Lange 2009)
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- **France (1876–96):** the level of education attainment had an adverse effect on fertility rates during France’s demographic transition, accounting for income per capita, the gender literacy gap, and mortality rates.  
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- **England (1580-1871)** Adverse effect of family size on children’s literacy. (Klemp-Weisdorf 2016)