The Demographic Transition

Ömer Özak

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Economic Growth and Comparative Development
Phases of Development: Standard of Living

- The Malthusian Epoch
Phases of Development: Standard of Living

- The Malthusian Epoch
- The Post-Malthusian Regime
Phases of Development: Standard of Living

- The Malthusian Epoch
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- The Modern Growth Regime
Phases of Development: Timeline of the Most Developed Economies

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(99.8%)

200K BP
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Fertility, mortality and population growth decline very rapidly
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- Transition to Modern Growth
Variation in years elapsed since the Onset of the Fertility Decline
The Demographic Transition – Definitions

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  - total number of live births per 1,000 in a population in a period

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- Total Fertility Rate (TFR)
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- **Total Fertility Rate (TFR)**
  - average number of children that would be born to a woman over her lifetime if
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- **Net reproduction rate (NRR)**
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Early Fertility Decline – Western Offshoots

<table>
<thead>
<tr>
<th>Period</th>
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<tbody>
<tr>
<td>1000-1500</td>
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Early Fertility Decline – Western Europe

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Late Fertility Decline – Latin America

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Late Fertility Decline – Asia

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The graph shows the growth rate over different periods for Africa.
The Demographic Transition in Western Europe: Total Fertility Rates
Mortality Decline Western Europe: 1730-1920
The Evolution of Total Fertility Rate across Regions, 1960-1999
Decline in infant mortality rates across regions, 1960-1999
The Demographic Transition
Evidence

Timing of the Demographic Transition and Current Income per Capita

coef = 1.3462847, (robust) se = .10852591, t = 12.41
Theories of the Demographic Transition

Becker, 1960
- The cost of raising children is primarily parental time, the rise in income increased the opportunity cost of raising children, leading to a reduction in fertility.
- The income elasticity of child quality is larger than that of quantity; the rise in income led to a substitution of child quality for quantity, further reducing fertility.

Becker and Lewis, JPE 1973
- The decline in child mortality enabled families to attain their desirable number of children with fewer births, leading to a reduction in fertility.
Theories of the Demographic Transition

- The Rise in Income (Becker, 1960)

The cost of raising children is primarily parental time. The rise in income increased the opportunity cost of raising children $\Rightarrow$ reduction in fertility (Becker, 1960).

The income elasticity of child quality is larger than that of quantity $\Rightarrow$ substitution of child quality for quantity $\Rightarrow$ reduction in fertility (Becker and Lewis, JPE 1973).

The Decline in Child Mortality

Decline in child mortality enabled families to attain their desirable number of children with lower number of births $\Rightarrow$ reduction in fertility.
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Overview

The Old-Age Security Hypothesis (Caldwell, 1976)

Children is a form of investment good (in the absence of access to financial markets). Development of financial markets reduced the demand for children as an investment good ⇒ reduction in fertility.

The Decline in the Gender Wage Gap (Galor-Weil, AER 1996)

The process of development decreased the gender wage gap. The rise in the relative wages of women increased the opportunity cost of raising children more than family income ⇒ reduction in fertility.
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The Rise in Human Capital Formation:

*(Galor-Weil, AER 2000)*

The rise in the industrial demand for human capital induced human capital formation. The presence of a budget constraint leads to a substitution of child quality for quantity, resulting in a reduction in fertility.
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The Rise in Income: Income and Fertility (Again!)

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- Child rearing is time-intensive
- Household’s Budget constraint

\[ y \tau n + c \leq y \]
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Equivalently

\[ c \leq y(1 - \tau n) \]

- \( 1 \equiv \) household’s time endowment
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- Equivalently

\[ c \leq y(1 - \tau n) \]

- \( 1 \equiv \) household’s time endowment
- \( (1 - \tau n) \equiv \) labor force participation
The Rise in Income: Income and Fertility (Again!)

- Child rearing is time-intensive
- Household’s Budget constraint

\[ y \tau n + c \leq y \]

- \( y \equiv \) household’s income
- \( c \equiv \) household’s consumption
- \( n \equiv \) household’s children
- \( \tau \equiv \) time cost per child
- \( y\tau \equiv \) opportunity cost of raising a child

Equivalently

\[ c \leq y(1 - \tau n) \]

- \( 1 \equiv \) household’s time endowment
- \( (1 - \tau n) \equiv \) labor force participation
- \( \tau n \equiv \) time spent raising children
Rise in Income: Optimal Choice

\[ U_0 c_0^n \tau \]

\[ U_1 c_1^n \tau \]

\[ U_2 c_2^n \tau \]

\[ U_3 c_3^n \tau \]

\[ \gamma^0 \]
Rise in Income: Optimal Choice

\[ n\tau \]

\[ U^0 \]

\[ y^0 \]
Rise in Income: Optimal Choice

\[ n\tau \]

\[ \tau n^0 \]

\[ C^0 \]

\[ y^0 \]

\[ U^0 \]
Rise in Income: Optimal Choice

\[ n\tau \]

\[ \tau n^0 \]

\[ c^0 \]

\[ y^0 \]

\[ U^0 \]

\[ c^1 \]

\[ y^1 \]
Rise in Income: Optimal Choice
Rise in Income: Optimal Choice

\[ n\tau \]

\[ U^0 \]

\[ U^1 \]

\[ c^0, c^1, y^0, y^1 \]
Rise in Income: Optimal Choice

\[ \begin{align*}
  &\text{Rise in Income: Optimal Choice} \\
  &\text{Theories} \quad \text{The Rise in Income}
\end{align*} \]
Rise in Income: Optimal Choice
Rise in Income: Optimal Choice

\[ n \tau \]

\[ \frac{\tau n^2}{n^2} \]

\[ U \]

\[ c^0 \quad c^1 \quad c^2 \quad y^0 \quad y^1 \quad y^2 \]
Rise in Income: Optimal Choice

\[ n\tau \]

\[ c^0 \quad c^1 \quad c^2 \quad y^0 \quad y^1 \quad y^2 \quad y^3 \]

\[ U^0 \quad U^1 \quad U^2 \]

\[ \frac{\delta n}{\delta n} \]

\[ \text{Ömer Özak} \]

The Demographic Transition

Growth & Comparative Development
Rise in Income: Optimal Choice

Theories

The Rise in Income

$\text{ Rise in Income: Optimal Choice}$

\[ c_n c_0 \]

\[ n \tau \]

\[ c^0 c^1 c^2 y^0 y^1 y^2 y^3 \]

\[ U^0 U^1 U^2 U^3 \]

\[ \frac{\partial n \tau}{\partial n} \]

\[ \frac{\partial n \tau}{\partial n} \]
Rise in Income: Optimal Choice

\[
\begin{align*}
\tau_0^c & \quad \tau_n^c \\
U_0 & \quad U_1 \\
\tau_0^n & \quad \tau_n^n \\
c^0 & \quad c^1 \quad c^2 \quad c^3 \\
Y_0 & \quad Y_1 \quad Y_2 \quad Y_3
\end{align*}
\]
The Rise in Income: Mechanism

- The rise in income generates two conflicting effects:

  1. An income effect:
     \[ y \tau n + c \leq y \]
     More income can be devoted to raising children operates towards \( n \uparrow \).

  2. A substitution effect:
     \[ y \tau n + c \leq y \]
     The opportunity cost of raising children increases operates towards \( n \downarrow \).
The rise in income generates two conflicting effects:

- An income effect:
  - More income can be devoted to raising children operates towards $n \uparrow$
  - The opportunity cost of raising children increases operates towards $n \downarrow$
The rise in income generates two conflicting effects:

- An income effect:
  \[ y\tau n + c \leq [y] \uparrow \]
- A substitution effect:
  More income can be devoted to raising children operates towards \( n \uparrow \)

The opportunity cost of raising children increases operates towards \( n \downarrow \)
The Rise in Income: Mechanism

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The rise in income generates two conflicting effects:

- An income effect:
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- An income effect:
  \[ y \tau n + c \leq [y] \uparrow \]
  
  More income can be devoted to raising children
  
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- A substitution effect:
The rise in income generates two conflicting effects:

- **An income effect:**
  \[ y_T n + c \leq [y] \uparrow \]

  - More income can be devoted to raising children
  - Operates towards \( n \uparrow \)

- **A substitution effect:**
  \[ \uparrow [y_T] n + c \leq y \]
The rise in income generates two conflicting effects:

- An income effect:
  \[ y_T n + c \leq [y] \uparrow \]
  
  - More income can be devoted to raising children
  - Operates towards \( n \uparrow \)

- A substitution effect:
  \[ \uparrow [y_T] n + c \leq y \]
  
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The Rise in Income: Mechanism

- The rise in income generates two conflicting effects:
  - An income effect:
    \[ y\tau n + c \leq [y] \uparrow \]
    - More income can be devoted to raising children
    - operates towards \( n \uparrow \)
  - A substitution effect:
    \[ \uparrow [y\tau]n + c \leq y \]
    - The opportunity cost of raising children increases
    - operates towards \( n \downarrow \)
The Rise in Income: Mechanism

- The substitution effect dominates at a higher level of income
The Rise in Income: Mechanism

- The substitution effect dominates at a higher level of income
- As income increases fertility declines
The Rise in Income: Mechanism

- The substitution effect dominates at a higher level of income
- As income increases fertility declines
- Fertility declines in the process of development
The Rise in Income - Theoretical Evaluation

- Preference-based theory (unattractive)
The Rise in Income - Theoretical Evaluation

- Preference-based theory (unattractive)
  - Innate bias against child quantity beyond a certain level of income - non-refutable
The Rise in Income - Theoretical Evaluation

- Preference-based theory (unattractive)
  - Innate bias against child quantity beyond a certain level of income - non-refutable
  - Non-robust (e.g., the class of homothetic preferences will not trigger a fertility decline)
The Rise in Income - Homothetic Preferences

Preferences:

\[ u = n^{\gamma} c^{(1-\gamma)} \]

\[ 0 < \gamma < 1 \]
The Rise in Income - Homothetic Preferences

Preferences:

\[ u = n^\gamma c^{(1-\gamma)} \quad 0 < \gamma < 1 \]

Budget constraint

\[ y_T n + c \leq y \]
The Rise in Income - Homothetic Preferences

- Preferences:
  \[ u = n^\gamma c^{(1-\gamma)} \quad 0 < \gamma < 1 \]

- Budget constraint
  \[ y\tau n + c \leq y \]

- Optimization: (fraction \( \gamma \) of income is spent on children and \( (1 - \gamma) \) on consumption)
The Rise in Income - Homothetic Preferences

- Preferences:
  \[ u = n^\gamma c^{(1-\gamma)} \quad 0 < \gamma < 1 \]

- Budget constraint
  \[ y \tau n + c \leq y \]

- Optimization: (fraction \( \gamma \) of income is spent on children and \( (1 - \gamma) \) on consumption)
  \[ y \tau n = \gamma y \]
  \[ c = (1 - \gamma)y \]
The Rise in Income - Homothetic Preferences

- Optimal number of children

\[ n = \frac{\gamma}{\tau} \]
The Rise in Income - Homothetic Preferences

- Optimal number of children
  \[ n = \frac{\gamma}{\tau} \]

- The rise in income has no effect on fertility, i.e.,
  \[ |\text{Income effect}| = |\text{Substitution effect}| \]
The Rise in Income - Homothetic Preferences

- Optimal number of children
  \[ n = \gamma / \tau \]

- The rise in income has no effect on fertility, i.e.,
  \[ |\text{Income effect}| = |\text{Substitution effect}| \]

- Fertility is unaffected by the process of development
The Rise in Income - Homothetic Preferences

\[ n \tau 
\]

\[ y^0 \]

\[ C \]
The Rise in Income - Homothetic Preferences

\[ U^0 \cdot \tau_0 = c_0 \cdot y_0 \]

\[ U^1 \cdot \tau_1 = c_1 \cdot y_1 \]

\[ U^2 \cdot \tau_2 = c_2 \cdot y_2 \]

\[ U^3 \cdot \tau_3 = c_3 \cdot y_3 \]
The Rise in Income - Homothetic Preferences

\[ c^0 \gamma \eta^0 \]

\[ U^0 \]
The Rise in Income - Homothetic Preferences

\[ U^0 = c^0 y^0 \]
\[ U^1 = c^1 y^1 \]

Diagram illustrates the relationship between consumption (c) and income (y) with an indifference curve (U) at different points (y^0, y^1) with constant consumption (c^0).
The Rise in Income - Homothetic Preferences

\[ \begin{align*}
U_0 & = c_0^{1-n}(1-\tau) y_0^n \\
U_1 & = c_1^{1-n}(1-\tau) y_1^n \\
U_2 & = c_2^{1-n}(1-\tau) y_2^n \\
U_3 & = c_3^{1-n}(1-\tau) y_3^n
\end{align*} \]
The Rise in Income - Homothetic Preferences

\[ c_1 = \gamma y_1 \]

\[ U_0 = \tau n^0 \]

\[ U_1 = \tau n^1 \]

\[ \tau n^1 = \tau n^0 \]

\[ c^0, c^1, y^0, y^1 \]
The Rise in Income - Homothetic Preferences

\[ c_0 \gamma_0 \tau_{n0} = c_1 \gamma_1 \tau_{n1} = c_2 \gamma_2 \tau_{n2} \]

The diagram illustrates the concept of homothetic preferences, where the indifference curves are normal to the budget constraint. The figure shows the effect of income changes on consumption: as income increases from \( y^0 \) to \( y^1 \) to \( y^2 \), consumption increases from \( c^0 \) to \( c^1 \) to \( c^2 \).
The Rise in Income - Homothetic Preferences

The diagram illustrates the concept of homothetic preferences in the context of income growth. The equations shown are:

\[ U_1 = \frac{\tau n_1}{\tau n_0} = U_0 \]

where

- \( n \) represents population
- \( \tau n \) is the rate of change of population
- \( U \) represents utility

The diagram shows different income levels \( y^0, y^1, y^2 \) associated with consumption levels \( c^0, c^1 \) and utility levels \( U^0, U^1, U^2 \), illustrating how utility remains constant as income and consumption change in a homothetic manner.
The Rise in Income - Homothetic Preferences

\[ \tau n^2 = \tau n^1 = \tau n^0 \]

\[ U^2 \quad U^1 \quad U^0 \]
The Rise in Income - Homothetic Preferences

\[ \tau n^2 = \tau n^1 = \tau n^0 \]

\[ U^2 \]

\[ U^1 \]

\[ U^0 \]
The Rise in Income - Homothetic Preferences

\[ \tau n^2 = \tau n^1 = \tau n^0 \]
The Rise in Income - Homothetic Preferences

\[\tau n^3 = \tau n^2 = \tau n^1 = \tau n^0\]
The Rise in Income - Homothetic Preferences

\[ \tau n^3 = \tau n^2 = \tau n^1 = \tau n^0 \]

\[ U^3 \]
\[ U^2 \]
\[ U^1 \]
\[ U^0 \]
Across countries that are similar in sociocultural characteristics (and thus in noneconomic factors that may affect fertility decisions), the timing of the fertility decline is inversely related to the level of income per capita.
The Rise in Income: Testable predictions

- Across countries that are similar in sociocultural characteristics (and thus in noneconomic factors that may affect fertility decisions), the timing of the fertility decline is inversely related to the level of income per capita.

- Within an economy, the number of (surviving) children across households is inversely related to their levels of income.
The Rise in Income: Refuting Cross Country Evidence

- Cross Section of Countries (1870-2000) - Income per worker was positively associated with fertility rates, accounting for mortality rates and education (Murtin 2013).
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Western Europe (1870s) The DT occurred among countries that differed significantly in their income per capita.
Simultaneous DT across European Countries that Differ in Income per Capita
The Rise in Income: Refuting Evidence from Individual Countries

- France (1876–96) Income per capita had a positive effect on fertility rates during France’s demographic transition, accounting for education, the gender literacy gap, and mortality rates (Murphy 2015)
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England (During the DT): The force associated with the rise in income would have led to an increase in fertility rates (Fernandez-Villaverde 2001).
The Rise in Income: Refuting Evidence from Individual Countries

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- **England (During the DT)**: The force associated with the rise in income would have led to an increase in fertility rates (Fernandez-Villaverde 2001)

- **England (1630s)** Reproductive success increases with income (Clark and Hamilton JEH 2006)
The Decline in Child Mortality - Main Hypothesis

Parents generate utility from the number of surviving children. A decline in child mortality permits parents to reach a given level of surviving children with lower fertility. The decline in mortality triggered the subsequent decline in fertility.
Parents generate utility from the number of surviving children
The Decline in Child Mortality - Main Hypothesis

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- Parents generate utility from the number of surviving children
- A decline in child mortality permits parents to reach a given level of surviving children with lower fertility
- The decline in mortality triggered the subsequent decline in fertility
The Decline in Mortality – Mechanism

\[ u = n \gamma c (1 - \gamma) \]

\[ c \equiv \text{household's consumption} \]
\[ n \equiv \text{household's surviving children} \]

\[ n_b = \theta n \]

\[ \theta \equiv \text{probability of a child to survive infancy} \]
\[ n_b \equiv \text{household's children born} \]
The Decline in Mortality – Mechanism

Preferences:

\[ u = n^\gamma c^{(1-\gamma)} \quad \text{for } 0 < \gamma < 1 \]
The Decline in Mortality – Mechanism

Preferences:

$$u = n^\gamma c^{(1-\gamma)}$$

$$0 < \gamma < 1$$

- $c \equiv$ household’s consumption
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Survival children
\[ n = \theta n^b \]
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\[ n = \theta n^b \]

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The Decline in Mortality – Mechanism

\[ y \tau_n + c \leq y \]

\[ y \equiv \text{household's income} \]
\[ c \equiv \text{household's consumption} \]
\[ \tau \equiv \text{time cost of raising a surviving child} \]
\[ y \tau \equiv \text{opportunity cost of raising a surviving child} \]
The Decline in Mortality – Mechanism

- Budget constraint

\[ y\tau n + c \leq y \]
The Decline in Mortality – Mechanism

- Budget constraint

\[ y \tau n + c \leq y \]

- \( y \equiv \) household’s income
The Decline in Mortality – Mechanism

- Budget constraint

\[ y \tau n + c \leq y \]

- \( y \equiv \) household’s income
- \( c \equiv \) household’s consumption
The Decline in Mortality – Mechanism

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- \( y\tau \equiv \) opportunity cost of raising a surviving child
The Decline in Mortality – Mechanism

- Optimization: (fraction $\gamma$ of income is spent on children and $(1 - \gamma)$ on consumption)
The Decline in Mortality – Mechanism

- Optimization: (fraction $\gamma$ of income is spent on children and $(1 - \gamma)$ on consumption)

\[ y_T n = \gamma y \]
\[ c = (1 - \gamma)y \]
The Decline in Mortality – Mechanism

- Optimization: (fraction $\gamma$ of income is spent on children and $(1 - \gamma)$ on consumption)
  \[ y \tau n = \gamma y \]
  \[ c = (1 - \gamma)y \]

- Optimal number of surviving children (NRR)
  \[ n = \frac{\gamma}{\tau} \]
The Decline in Mortality – Mechanism

- Optimization: (fraction $\gamma$ of income is spent on children and $(1 - \gamma)$ on consumption)

$$y\tau n = \gamma y$$
$$c = (1 - \gamma)y$$

- Optimal number of surviving children (NRR)

$$n = \gamma / \tau$$

- Optimal fertility (# of successful pregnancies - TFR)

$$n^b = n / \theta = \gamma / (\tau \theta)$$
The Decline in Mortality – Testable Predictions

- Child mortality rate, $(1 - \theta)$, has a positive effect on TFR
The Decline in Mortality – Testable Predictions

- Child mortality rate, $(1 - \theta)$, has a positive effect on TFR
- Child mortality rate, $(1 - \theta)$, has no effect on NRR
Worldwide: NRR and TFR plummet jointly during the demographic transition (Lehr 2009). But the theory does not predict a decline in NRR
The Decline in Child Mortality

- Worldwide: NRR and TFR plummet jointly during the demographic transition (Lehr 2009). But the theory does not predict a decline in NRR.

- NRR does not decline unless:
  - There exists a precautionary demand for children.
  - RA with respect to fertility > RA with respect to consumption.
  - Replacement fertility is insignificant (empirical estimates 0.2–0.6).
  - Resources saved from investment in non-surviving children are not channeled towards higher fertility.
**The Decline in Child Mortality**

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The Decline in Mortality and Fertility - Evidence
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The Decline in Child Mortality – Challenges to the Theory

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- Western Europe: No change in the patterns of mortality decline at the time of the sharp decline in fertility

- England: The decline in mortality started in the 1730s (140 years before the fertility decline) and was accompanied by a steady increase in fertility rates until 1800
France (1876–96): Mortality rate had no effect on fertility during France’s demographic transition, accounting for education, income, and the gender literacy gap. (Murphy 2009)
The Decline in Mortality: Refuting Evidence from Individual Countries

- **France (1876–96):** Mortality rate had no effect on fertility during France’s demographic transition, accounting for education, income, and the gender literacy gap. (Murphy 2009)

- **England (1861–1951):** The force associated with the decline in child mortality would have led to an increase in fertility rates (Fernandez-Villaverde 2001; Doepke 2005)
The Old-Age Security Hypothesis

Children is a form of investment good (in the absence of capital markets).

The development of financial markets reduced the demand for children for investment purposes and triggered a decline in fertility.
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The Old-Age Security Hypothesis - Challenges to the Theory

- The decline in the importance of old-age support is unlikely to be a major force behind the significant reduction in fertility – at a rate of 30–50% – during the demographic transition:
The Old-Age Security Hypothesis - Challenges to the Theory

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  - Rare examples in nature of offspring that support their parents in old age
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- Institutions supporting individuals in their old age were formed well before the demographic transition
  - England (16th century) Parents did not rely on support from children in their old age (Pelling and Smith 1991)
- Prior to the demographic transition, richer individuals who presumably had better access to financial markets, had larger number of surviving children
The Decline in the Gender Wage Gap

The inevitable rise in the relative wages of women in the process of development increases the opportunity cost of raising children more than family income \[ \Rightarrow \] reduction in fertility.
The Decline in the Gender Wage Gap

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The Decline in the Gender Wage Gap

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  - increases the opportunity cost of raising children more than family income
The Decline in the Gender Wage Gap

- The inevitable rise in the relative wages of women in the process of development
  - increases the opportunity cost of raising children more than family income
  ⇒ reduction in fertility
Mechanism: I. Development and Women’s Wages

Female-Biased Technical change

Mechanization and advanced technologies have complemented mental tasks more than physical tasks.

Women have physiological comparative advantage in mental (rather than physical) tasks.

⇒ The process of development has (inevitably) increased the productivity of women relative to men:

\[ \frac{w_F}{w_M} \uparrow \]

\( w_F \equiv \) women’s wages

\( w_M \equiv \) men’s wages
Mechanism: I. Development and Women’s Wages

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\[
\text{Economic Development } \implies \left( \frac{w^F}{w^M} \right) \uparrow
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\text{Economic Development } \implies \left( \frac{w^F}{w^M} \right) \uparrow
\]

- \( w^F \equiv \text{women’s wages} \)
Mechanism: I. Development and Women’s Wages

- Female-Biased Technical change
  - Mechanization and advanced technologies have complemented mental tasks more than physical tasks
  - Women have physiological comparative advantage in mental (rather than physical) tasks

⇒ The process of development has (inevitably) increased the productivity of women relative to men:

\[ \text{Economic Development} \implies \left( \frac{w^F}{w^M} \right) \uparrow \]

- \( w^F \equiv \) women’s wages
- \( w^M \equiv \) men’s wages
Evolution of the Gender Earnings Ratio - US
Evolution of the Gender Literacy Gap - England
Mechanism: Women’s Relative Wages and Fertility

Child rearing is time-intensive. Women are the prime care-takers engaged in child rearing. A budget constraint arises if only women raise children:

\[ w_F \tau_n + c \leq w_M + w_F \equiv \text{household's income} \]
\[ c \equiv \text{household's consumption} \]
\[ n \equiv \text{household's (surviving) children} \]
\[ \tau \equiv \text{time cost per child} \]
\[ w_F \tau \equiv \text{opportunity cost of raising a child} \]
Mechanism: Women’s Relative Wages and Fertility

- Child rearing is time-intensive
Mechanism: Women’s Relative Wages and Fertility

- Child rearing is time-intensive
- Women are the prime care-takers engaged in child rearing
Mechanism: Women’s Relative Wages and Fertility

- Child rearing is time-intensive
- Women are the prime care-takers engaged in child rearing
- Budget constraint (if only women raise children)

$$w^F \tau n + c \leq w^M + w^F$$
Mechanism: Women’s Relative Wages and Fertility

- Child rearing is time-intensive
- Women are the prime care-takers engaged in child rearing
- Budget constraint (if only women raise children)

\[ w^F \tau n + c \leq w^M + w^F \]

- \( w^F + w^M \equiv \) household’s income
Mechanism: Women’s Relative Wages and Fertility

- Child rearing is time-intensive
- Women are the prime care-takers engaged in child rearing
- Budget constraint (if only women raise children)

\[ w^F \tau n + c \leq w^M + w^F \]

- \( w^F + w^M \equiv \text{household’s income} \)
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- Budget constraint (if only women raise children)

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- \( w^F + w^M \equiv \) household’s income
- \( c \equiv \) household’s consumption
- \( n \equiv \) household’s (surviving) children
- \( \tau \equiv \) time cost per child
- \( w^F \tau \equiv \) opportunity cost of raising a child
Mechanism: Women’s Relative Wages and Fertility

\[ \frac{2}{\tau} \]

\[ \frac{1}{\tau} \]

\[ w^M + w^F \]
Mechanism: Women’s Relative Wages and Fertility
Mechanism: Women’s Relative Wages and Fertility

\[ n^0 \]

\[ c^0 w^M + w^F \]

\[ U^0 \]
Mechanism: Women’s Relative Wages and Fertility

\[ n = \frac{2}{\tau} \]

\[ n^0 = \frac{1}{\tau} \]

\[ c^0 \]

\[ w^M + w^F \]

\[ w^M + w^{F'} \]
Mechanism: Women’s Relative Wages and Fertility

\[ n^0 \]

\[ \frac{1}{\tau} \]

\[ \frac{2}{\tau} \]

\[ n^0 \]

\[ \tau \]

\[ U^0 \]

\[ U^1 \]

\[ c^0 \]

\[ c^0 \]

\[ w^M + w^F \]

\[ w^M + w^F' \]
Mechanism: Women’s Relative Wages and Fertility

\[ c_1 \tau w_M + w_F' \]

\[ n \]

\[ c_0 \tau w_M + w_F \]

\[ n^0 \]

\[ n^1 \]
Mechanism: Women’s Relative Wages and Fertility

The rise in women’s wages, $w_F$, generates two conflicting effects:

**An income effect:**

$$w_F \tau_n + c \leq w_M + \left(w_F\right)$$

More income for raising children $\Rightarrow$ operates towards $n \uparrow$

**A substitution effect:**

$$\uparrow \left[w_F \tau_n + c \leq w_M + w_F\right]$$

Opportunity cost of children increases $\Rightarrow$ operates towards $n \downarrow$

A rise in men’s wages generate only an income effect

$$w_F \tau_n + c \leq \left[w_M\right]$$
Mechanism: Women’s Relative Wages and Fertility

- The rise in women’s wages, $w^F$, generates two conflicting effects:
  
  - An income effect:
    
    \[ w^F \tau_n + c \leq w^M + [w^F] \uparrow \Rightarrow \text{operates towards } n \uparrow \]
    
    More income for raising children = \Rightarrow operates towards $n \uparrow$

  - A substitution effect:
    
    \[ \uparrow [w^F \tau_n] + c \leq w^M + w^F \Rightarrow \text{operates towards } n \downarrow \]
    
    Opportunity cost of children increases = \Rightarrow operates towards $n \downarrow$

A rise in men’s wages generate only an income effect

\[ w^F \tau_n + c \leq [w^M] \uparrow + [w^F] \]

\[ \Rightarrow \text{operates towards } n \downarrow \]
Mechanism: Women’s Relative Wages and Fertility

The rise in women’s wages, $w^F$, generates two conflicting effects:

- An income effect:
  \[ w^F \tau n + c \leq w^M + [w^F] \uparrow \]

- Opportunity cost of children increases

A rise in men’s wages generates only an income effect:

\[ w^F \tau n + c \leq w^M \]

\[ \uparrow \]

\[ \text{Or} \]

\[ w^M \text{ generates only an income effect.} \]
Mechanism: Women’s Relative Wages and Fertility

- The rise in women’s wages, $w^F$, generates two conflicting effects:
  - An income effect:
    $$w^F \tau_n + c \leq w^M + [w^F] \uparrow$$
    - More income for raising children
  - A substitution effect:
    $$[w^F \tau_n] + c \leq w^M + w^F$$
    - Opportunity cost of children increases
- A rise in men’s wages generate only an income effect
  $$w^F \tau_n + c \leq [w^M] \uparrow$$
Mechanism: Women’s Relative Wages and Fertility

- The rise in women’s wages, $w^F$, generates two conflicting effects:
  - An income effect:
    $$w^F \tau n + c \leq w^M + [w^F] \uparrow$$
    - More income for raising children $\implies$ operates towards $n \uparrow$
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Mechanism: Women’s Relative Wages and Fertility

- The rise in women’s wages, $w^F$, generates two conflicting effects:
  - An income effect:
    \[
    w^F n + c \leq w^M + [w^F] \uparrow
    \]
    - More income for raising children $\Rightarrow$ operates towards $n \uparrow$
  - A substitution effect:
    \[
    \uparrow [w^F n + c] \leq w^M + w^F
    \]
Mechanism: Women’s Relative Wages and Fertility

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Mechanism: Women’s Relative Wages and Fertility

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Mechanism: Women’s Relative Wages and Fertility

- The rise in women’s wages, $w^F$, generates two conflicting effects:
  - An income effect:
    \[ w^F \tau n + c \leq w^M + [w^F] \uparrow \]
    - More income for raising children \(\Rightarrow\) operates towards \(n \uparrow\)
  - A substitution effect:
    \[ \uparrow [w^F \tau] n + c \leq w^M + w^F \]
    - Opportunity cost of children increases \(\Rightarrow\) operates towards \(n \downarrow\)
- A rise in men’s wages generate only an income effect
  \[ w^F \tau n + c \leq [w^M] \uparrow + [w^F] \]
The Decline in the Gender Wage Gap

- If women work and raise children, an increase in $w^F$ increases the opportunity cost of raising children more than family income,
The Decline in the Gender Wage Gap

- If women work and raise children, an increase in $w^F$ increases the opportunity cost of raising children more than family income, i.e.,

$$w^F \uparrow \implies |\text{Income effect}| < |\text{Substitution effect}|$$

$$\implies n \downarrow \text{ (even if preferences are homothetic)}$$
The Decline in the Gender Wage Gap

- If women work and raise children, an increase in $w^F$ increases the opportunity cost of raising children more than family income, i.e.,

  
  \[
  w^F \uparrow \implies |\text{Income effect}| < |\text{Substitution effect}|
  \]

  \[
  \implies n \downarrow \text{ (even if preferences are homothetic)}
  \]

- A rise in men’s wages generate only an income effect
The Decline in the Gender Wage Gap

- If women work and raise children, an increase in $w^F$ increases the opportunity cost of raising children more than family income, i.e.,

$$w^F \uparrow \implies |\text{Income effect}| < |\text{Substitution effect}|$$

$$\implies n \downarrow \quad \text{(even if preferences are homothetic)}$$

- A rise in men’s wages generate only an income effect

$$w^M \uparrow \implies n \uparrow$$
Women’s Relative Wages and Fertility - US
Women’s Relative Wages and Fertility - Evidence

- US (1970s): $w^F \uparrow \implies n \downarrow$ & $w^M \uparrow \implies n \uparrow$ (Heckman and Walker ECT 79)

- Sweden’s demographic transition: $(w^F/w^M) \uparrow \implies n \downarrow$ (Schultz 1985)

- France (1876–1896): reduction in the gender literacy gap had an adverse effect on fertility, accounting for income per capita, educational attainment, and mortality rates (Murphy 2015)
Women’s Relative Wages and Fertility - Evidence

- **US (1970s):** \( w^F \uparrow \implies n \downarrow \) & \( w^M \uparrow \implies n \uparrow \) (Heckman and Walker ECT 79)

- Sweden’s demographic transition: \( (w^F/w^M) \uparrow \implies n \downarrow \) (Schultz 1985)
Women’s Relative Wages and Fertility - Evidence

- **US (1970s):** $w^F \uparrow \implies n \downarrow \ & \ w^M \uparrow \implies n \uparrow$ (Heckman and Walker ECT 79)

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- **France (1876–1896):** reduction in the gender literacy gap had an adverse effect on fertility, accounting for income per capita, educational attainment, and mortality rates (Murphy 2015)
The Rise in the Demand for Human Capital - Main Thesis

The acceleration in the rate of technological progress in the 2nd phase of industrialization increased the demand for human capital. Education enabled individuals to cope with a rapidly changing technological environment. The rise in the demand for human capital induced a substitution of quality for quantity of children triggering a demographic transition. \[\Rightarrow \text{reduction in fertility}\]
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The Rise in the Demand for Human Capital - Main Thesis

- The acceleration in the rate of technological progress in the 2nd phase of industrialization increased the demand for human capital
  - education enabled individuals to cope with a rapidly changing technological environment

- The rise in the demand for human capital induced a substitution of quality for quantity of children triggering a demographic transition
  \[\Rightarrow\] reduction in fertility
The Model - Preferences

\[ u = (1 - \gamma) \ln c + \gamma[\ln n + \beta \ln h] \]
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- \( c \equiv \) consumption
The Model - Preferences

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- \( c \equiv \) consumption
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The Model - Preferences

\[ u = (1 - \gamma) \ln c + \gamma[\ln n + \beta \ln h] \]

- \( c \equiv \) consumption
- \( n \equiv \) (surviving) children
- \( h \equiv \) quality (human capital) of each child
The Model - Preferences

\[ u = (1 - \gamma) \ln c + \gamma [\ln n + \beta \ln h] \]

- \( c \equiv \text{consumption} \)
- \( n \equiv \text{(surviving) children} \)
- \( h \equiv \text{quality (human capital) of each child} \)
- \( \beta \equiv \text{degree of preference for child quality} \)


The Model - Preferences

\[ u = (1 - \gamma) \ln c + \gamma [\ln n + \beta \ln h] \]

- \( c \equiv \) consumption
- \( n \equiv \) (surviving) children
- \( h \equiv \) quality (human capital) of each child
- \( \beta \equiv \) degree of preference for child quality; \( \beta < 1 \)
The Model - Budget Constraint

\[ yn(\tau^q + \tau^e e) + c \leq y \]
The Model - Budget Constraint

\[ yn(\tau^q + \tau^e e) + c \leq y \]

- \( y \equiv \) household potential income
The Model - Budget Constraint

\[ yn(\tau^q + \tau^e e) + c \leq y \]

- \( y \equiv \) household potential income
- \( \tau^q \equiv \) fraction of the household’s unit-time endowment required to raise a child, regardless of quality

\[ \tau^e e \equiv \text{time cost of raising a child with a level of education (quality)} \]

\[ y^2 \equiv \text{opportunity cost of raising a child with quality} \]
The Model - Budget Constraint

\[ yn(\tau^q + \tau^e e) + c \leq y \]

- \( y \equiv \) household potential income
- \( \tau^q \equiv \) fraction of the household’s unit-time endowment required to raise a child, regardless of quality
- \( \tau^e \equiv \) fraction of the household’s unit-time endowment required for each unit of education per child
The Model - Budget Constraint

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- \( \tau^e \equiv \) fraction of the household’s unit-time endowment required for each unit of education per child
- \( (\tau^q + \tau^e e) \equiv \) time cost of raising a child with a level of education (quality)
The Theories of Human Capital Formation

The Model - Budget Constraint

\[ yn(\tau^q + \tau^e e) + c \leq y \]

- \( y \equiv \) household potential income
- \( \tau^q \equiv \) fraction of the household’s unit-time endowment required to raise a child, regardless of quality
- \( \tau^e \equiv \) fraction of the household’s unit-time endowment required for each unit of education per child
- \( (\tau^q + \tau^e e) \equiv \) time cost of raising a child with a level of education (quality) \( e \)
- \( y(\tau^q + \tau^e e) \equiv \) opportunity cost of raising a child with quality \( e \)
The Model - Human Capital Formation

\[ h = h(e, g) \]
The Model - Human Capital Formation

\[ h = h(e, g) \]

- \( h_e(e, g) > 0 \) & \( h_{ee}(e, g) < 0 \)
The Model - Human Capital Formation

\[ h = h(e, g) \]

- \( h_e(e, g) > 0 \) & \( h_{ee}(e, g) < 0 \)
- HC is increasing (at decreasing rates) in the parental time investment in the education of the child

\[ h(0, g) > 0 \text{ & } \lim_{e \to 0} h(e, g) = \infty \]
\[ \lim_{e \to \infty} h(e, g) = 0 \]

Basic level of human capital & interior solution
The Model - Human Capital Formation

\[ h = h(e, g) \]

- \( h_e(e, g) > 0 \) & \( h_{ee}(e, g) < 0 \)
  - HC is increasing (at decreasing rates) in the parental time investment in the education of the child

- \( h_g(e, g) < 0 \) & \( h_{gg}(e, g) > 0 \)
**The Model - Human Capital Formation**

\[ h = h(e, g) \]

- \( h_e(e, g) > 0 \) & \( h_{ee}(e, g) < 0 \)
  - HC is increasing (at decreasing rates) in the parental time investment in the education of the child

- \( h_g(e, g) < 0 \) & \( h_{gg}(e, g) > 0 \)
  - HC is decreasing in the rate of technological progress (obsolescence of HC in a changing technological environment)
The Theories of Human Capital Formation

The Rise in Human Capital Formation

The Model - Human Capital Formation

\[ h = h(e, g) \]

- \( h_e(e, g) > 0 \) \& \( h_{ee}(e, g) < 0 \)
  - HC is increasing (at decreasing rates) in the parental time investment in the education of the child

- \( h_g(e, g) < 0 \) \& \( h_{gg}(e, g) > 0 \)
  - HC is decreasing in the rate of technological progress (obsolescence of HC in a changing technological environment)

- \( h_{eg}(e, g) > 0 \)
The Model - Human Capital Formation

\[ h = h(e, g) \]

- \( h_e(e, g) > 0 \) & \( h_{ee}(e, g) < 0 \)
  - HC is increasing (at decreasing rates) in the parental time investment in the education of the child

- \( h_g(e, g) < 0 \) & \( h_{gg}(e, g) > 0 \)
  - HC is decreasing in the rate of technological progress (obsolescence of HC in a changing technological environment)

- \( h_{eg}(e, g) > 0 \)
  - Education lessens the obsolescence of HC in a changing technological environment
The Model - Human Capital Formation

\[ h = h(e, g) \]

- \( h_e(e, g) > 0 \) & \( h_{ee}(e, g) < 0 \)
  - HC is increasing (at decreasing rates) in the parental time investment in the education of the child

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\[ h = h(e, g) \]

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  - Education lessens the obsolescence of HC in a changing technological environment

- \( h(0, g) > 0 \) & \( \lim_{e \to 0} h(e, g) = \infty; \lim_{e \to \infty} h(e, g) = 0 \)
  - Basic level of human capital & interior solution
The Model - Optimization

\[
\begin{align*}
    \{n, e, c\} &= \text{arg max} \left\{ \gamma \ln n + \beta \ln h(e, g) + (1 - \gamma) \ln y \right\} \\
    &= \{n, e\} \text{ since } c = y \left[ 1 - n(\tau q + \tau e) \right] \\
    &\iff \{n, e\} = \text{arg max} \left\{ \gamma \ln n + \beta \ln h(e, g) + (1 - \gamma) \ln y \right\}
\end{align*}
\]

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Theories  The Rise in Human Capital Formation

Omer Ozak  The Demographic Transition  Growth & Comparative Development
The Model - Optimization

\[ \{n, e, c\} = \operatorname{arg\,max} \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln c \]

\[ \text{s.t. } yn(\tau^q + \tau^e e) + c \leq y \]
The Model - Optimization

\[ \{n, e, c\} = \operatorname{arg\ max} \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln c \]
\[ \text{s.t.} \quad yn(\tau^q + \tau^e e) + c \leq y \]

since \( c = y [1 - n(\tau^q + \tau^e e)] \)
The Model - Optimization

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\{n, e, c\} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln c \\
\text{s.t.} \quad yn(\tau^q + \tau^e e) + c \leq y
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The Model - Optimization

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\]
\[
\text{s.t. } yn(\tau^q + \tau^e e) + c \leq y
\]

since \( c = y[1 - n(\tau^q + \tau^e e)] \) \( \iff \)

\[
\{ n, e \} = \arg \max \gamma[\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y[1 - n(\tau^q + \tau^e e)]
\]
\[ \{n, e\} = \text{arg max } \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y[1 - n(\tau^q + \tau^e e)] \]
Optimization

\[ \{ n, e \} = \text{arg max } \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y [1 - n(\tau^q + \tau^e e)] \]

with respect to \( n \):
Optimization

\[ \{n, e\} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y[1 - n(\tau^q + \tau^e e)] \]

with respect to \( n \):

\[ \frac{\gamma}{n} = \frac{(1 - \gamma)y(\tau^q + \tau^e e)}{y[1 - n(\tau^q + \tau^e e)]} \]
\{n, e\} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y [1 - n(\tau^q + \tau^e e)]

with respect to \(n\):

\[
\frac{\gamma}{n} = \frac{(1 - \gamma)y(\tau^q + \tau^e e)}{y[1 - n(\tau^q + \tau^e e)]}
\]

\[
\gamma [1 - n(\tau^q + \tau^e e)] = (1 - \gamma)(\tau^q + \tau^e e)n
\]
Optimization

\[ \{ n, e \} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y[1 - n(\tau^q + \tau^e e)] \]

- with respect to \( n \):

\[
\frac{\gamma}{n} = \frac{(1 - \gamma)y(\tau^q + \tau^e e)}{y[1 - n(\tau^q + \tau^e e)]}
\]

\[ \gamma[1 - n(\tau^q + \tau^e e)] = (1 - \gamma)(\tau^q + \tau^e e)n \]

\[ n(\tau^q + \tau^e e) = \gamma \]
Theories

The Rise in Human Capital Formation

Optimization

\[ \{ n, e \} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y[1 - n(\tau^q + \tau^e e)] \]
Optimization

\[
\{n, e\} = \arg\max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y[1 - n(\tau^q + \tau^e e)]
\]

with respect to \(e\):

\[
\frac{\gamma \beta h(e, g)}{h(e, g)} = \frac{(1 - \gamma)yn\tau^e}{y[1 - n(\tau^q + \tau^e e)]}
\]
Optimization

\[ \{n, e\} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y[1 - n(\tau^q + \tau^e e)] \]

with respect to \( e \):

\[ \frac{\gamma \beta h_e(e, g)}{h(e, g)} = \frac{(1 - \gamma)yn\tau^e}{y[1 - n(\tau^q + \tau^e e)]} \]

since \( n(\tau^q + \tau^e e) = \gamma \)
Theories

The Rise in Human Capital Formation

Optimization

\[ \{n, e\} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y[1 - n(\tau^q + \tau^e e)] \]

- with respect to \(e\):

\[ \frac{\gamma \beta h_e(e, g)}{h(e, g)} = \frac{(1 - \gamma)yn\tau^e}{y[1 - n(\tau^q + \tau^e e)]} \]

- since \(n(\tau^q + \tau^e e) = \gamma\)

\[ \frac{\gamma \beta h_e(e, g)}{h(e, g)} = n\tau^e \]
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- since \( n(\tau^q + \tau^e e) = \gamma \)

\[
\frac{\gamma \beta h_e(e, g)}{h(e, g)} = n\tau^e \quad \Rightarrow \quad \frac{\beta h_e(e, g)}{h(e, g)} = \frac{\tau^e}{(\tau^q + \tau^e e)}
\]
Optimization

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\{ n, e \} = \arg \max \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln y [1 - n(\tau^q + \tau^e e)]
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- with respect to \( e \):

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\frac{\gamma \beta h_e(e, g)}{h(e, g)} = \frac{(1 - \gamma)yn\tau^e}{y[1 - n(\tau^q + \tau^e e)]}
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- since \( n(\tau^q + \tau^e e) = \gamma \)

\[
\frac{\gamma \beta h_e(e, g)}{h(e, g)} = \tau^e = \frac{\beta h_e(e, g)}{h(e, g)} = \frac{\tau^e}{(\tau^q + \tau^e e)}
\]

\[\varepsilon_{h,e} \equiv \frac{h_e(e, g)e}{h(e, g)} = \frac{1}{\beta} \frac{\tau^e e}{(\tau^q + \tau^e e)} \equiv \sigma(e; \beta, \tau^q, \tau^e)\]
Optimal education choice

\[ \varepsilon_{e,h,\sigma(e)} \]
Optimal education choice

\[ \varepsilon_{e,h}, \sigma(e) \]

\[ \sigma(e; \beta, \tau^q, \tau^e) = \frac{1}{\beta} \frac{\tau^e}{(\tau^q + \tau^e)} \]
Optimal education choice

\[ \varepsilon_{e,h}, \sigma(e) \]

\[ \sigma(e; \beta, \tau^q, \tau^e) = \frac{1}{\beta} \frac{\tau^e e}{(\tau^q + \tau^e e)} \]

\[ \varepsilon_{e,h} = \frac{h(e,g)e}{h(e,g)} \]
So,
Optimization

So,

\[ n = \gamma / (\tau^q + \tau^e e) \]

\[ \tau^e h(e, g) = \beta h_e(e, g)(\tau^q + \tau^e e) \]
Optimization

So,

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Optimization

So,

\[ n = \frac{\gamma}{(\tau^q + \tau^e e)} \]

\[ \tau^e h(e, g) = \beta h_e(e, g)(\tau^q + \tau^e e) \]

\[ e = e(g, \beta, \tau^e, \tau^q), \]

\[ n = \frac{\gamma}{[\tau^q + \tau^e e(g, \beta, \tau^e, \tau^q)]} \]
Testable Predictions - Investment in Quality

The optimal level of investment in child quality increases if:

\[
\frac{\partial e(g, \beta, \tau_e, \tau_q)}{\partial g} > 0
\]

Preferences for child quality are higher
\[
\frac{\partial e(g, \beta, \tau_e, \tau_q)}{\partial \beta} > 0
\]

The cost of raising a child (regardless of quality) increases
\[
\frac{\partial e(g, \beta, \tau_e, \tau_q)}{\partial \tau_q} > 0
\]

The cost of educating a child decreases
\[
\frac{\partial e(g, \beta, \tau_e, \tau_q)}{\partial \tau_e} < 0
\]
Testable Predictions - Investment in Quality

The optimal level of investment in child quality increases if:

- The technological environment changes more rapidly
  \[ \frac{\partial e(g, \beta, \tau^e, \tau^q)}{\partial g} > 0 \]
- Preferences for child quality are higher
  \[ \frac{\partial e(g, \beta, \tau^e, \tau^q)}{\partial \beta} > 0 \]
- The cost of raising a child (regardless of quality) increases
  \[ \frac{\partial e(g, \beta, \tau^e, \tau^q)}{\partial \tau^q} > 0 \]
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- The cost of educating a child decreases
  \[ \frac{\partial e(g, \beta, \tau^e, \tau^q)}{\partial \tau^e} < 0 \]
Testable Predictions - Investment in Quantity

The optimal number of children decreases if:

- The technological environment changes more rapidly: \( \frac{\partial n}{\partial g} < 0 \)
- Preferences for child quality are higher: \( \frac{\partial n}{\partial \beta} < 0 \)
- The cost of raising a child (regardless of quality) increases: \( \frac{\partial n}{\partial \tau_q} < 0 \)
- The cost of educating a child increases and the elasticity of child quality with respect to the cost of child quality is smaller than one in absolute value: \( \frac{\partial n}{\partial \tau_e} < 0 \) if \( \frac{\partial e}{\partial \tau_e} \left( \frac{\tau_e}{e} \right) > -1 \)
Testable Predictions - Investment in Quantity

The optimal number of children decreases if:

- The technological environment changes more rapidly
  \[ \frac{\partial n}{\partial g} < 0 \]

- Preferences for child quality are higher
  \[ \frac{\partial n}{\partial \beta} < 0 \]

- The cost of raising a child (regardless of quality) increases
  \[ \frac{\partial n}{\partial \tau_q} < 0 \]

- The cost of educating a child increases and the elasticity of child quality with respect to the cost of child quality is smaller than one in absolute value
  \[ \frac{\partial n}{\partial \tau_e} < 0 \text{ if } \left| \frac{\partial e}{\partial \tau_e} \right| > 1 \]
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  \[ \frac{\partial n}{\partial \tau^e} < 0 \text{ if } \left[ \frac{\partial e}{\partial \tau^e} \right] \frac{\tau^e}{e} > -1 \]
Human Capital Formation and the DT - England
Growth Rates 1870-1913 and DT

Theories

The Rise in Human Capital Formation

The Demographic Transition

Growth & Comparative Development
Supporting Evidence: Cross-Country Evidence

- Cross Section of Countries (1870-2000) - educational attainment has been negatively associated with fertility, accounting for income per worker and mortality rates (Murtin 2013).
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- Cross Section of Countries (1960-1999): adverse effect on net fertility of an increase in productivity in advanced stages of development, when education demand dominates (Lehr 2009).
US (1910s): Eradication of hookworm – a positive shock to the return to child quality - had an adverse effect on fertility (Bleakley-Lange 2009)
Supporting Evidence: Evidence from Individual Countries

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- **England (1580-1871)** Adverse effect of family size on children’s literacy. (Klemp-Weisdorf 2016)