

# The Malthusian Hypothesis

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Economic Growth and Comparative Development

## Phases of Development: Standard of Living

- The Malthusian Epoch

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- The Post-Malthusian Regime

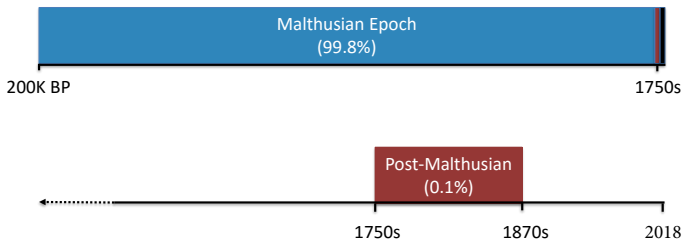
## Phases of Development: Standard of Living

- The Malthusian Epoch
- The Post-Malthusian Regime
- The Modern Growth Regime

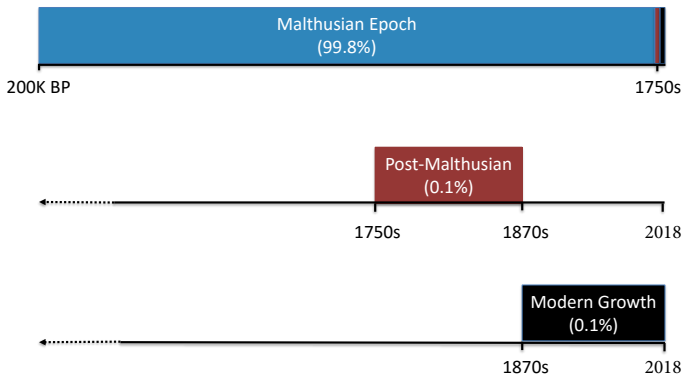
## Phases of Development: Timeline of the Most Developed Economies



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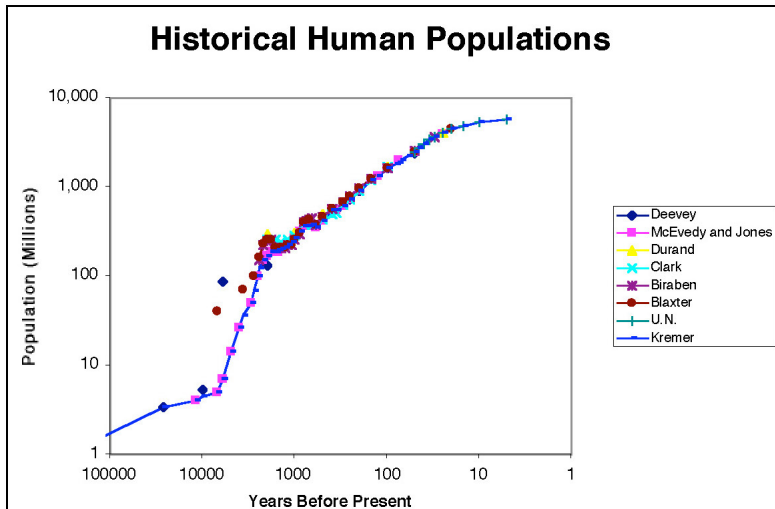
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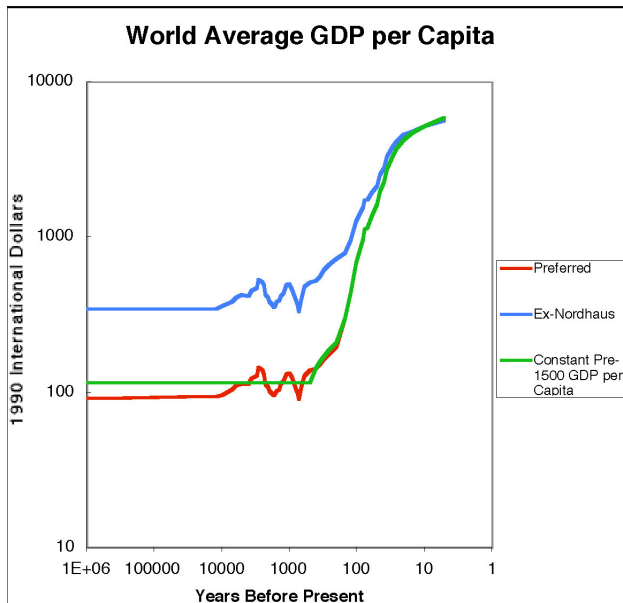
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- Technologically advanced & land-rich economies:
  - Higher population density
  - Similar levels of income per-capita in the long-run

## World Population 100,000 BP–1950CE



## World GDP per capita 100,000 BP–1950CE

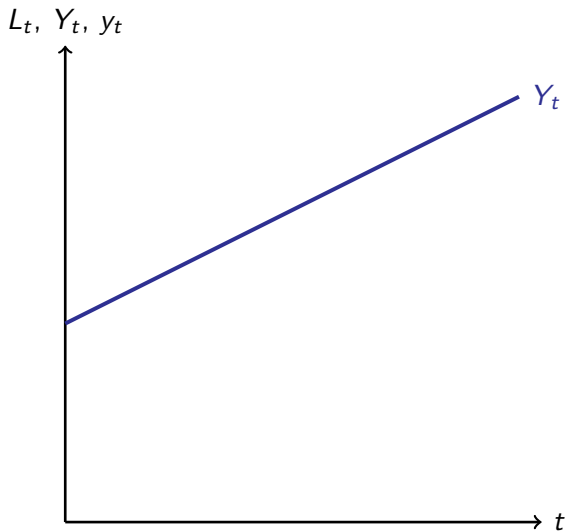


## Malthus' Theory

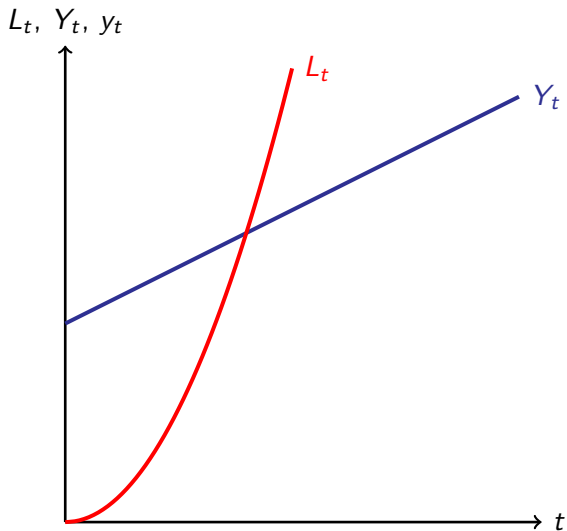
- Population and income growth

*"I think I may make fairly two postulata. First, that food is necessary to the existence of man. Secondly, that the passion between the sexes is necessary and will remain nearly in its present state ... Assuming then my postulata as granted, I say, that the power of population is infinitely greater than the power in the earth to produce subsistence for man. Population, when unchecked, increases in a geometrical ratio. Subsistence increases only in an arithmetical ratio. A slight acquaintance with numbers will show the immensity of the first power in comparison of the second. By the law of our nature which makes food necessary to the life of man, the effects of these two unequal powers must be kept equal. This implies a strong and constantly operating check on population from the difficulty of subsistence. This difficulty must fall somewhere and must necessarily be severely felt by a large portion of mankind...."*

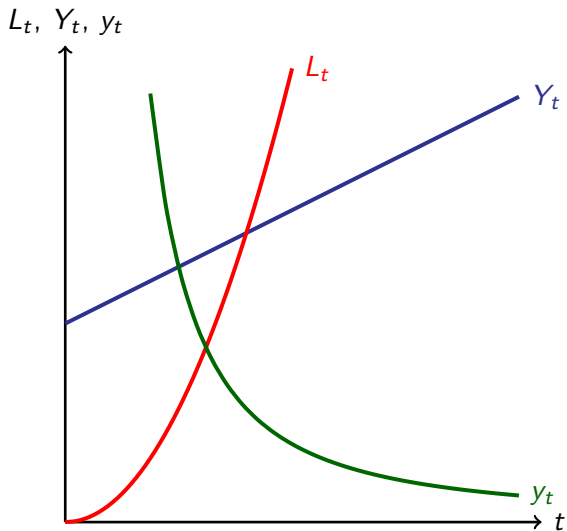
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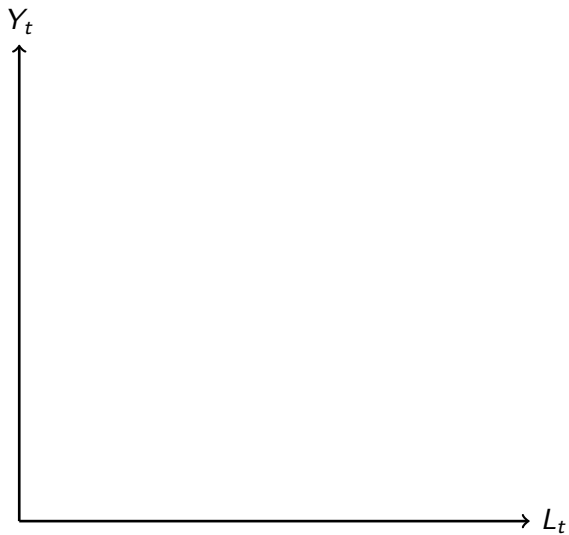


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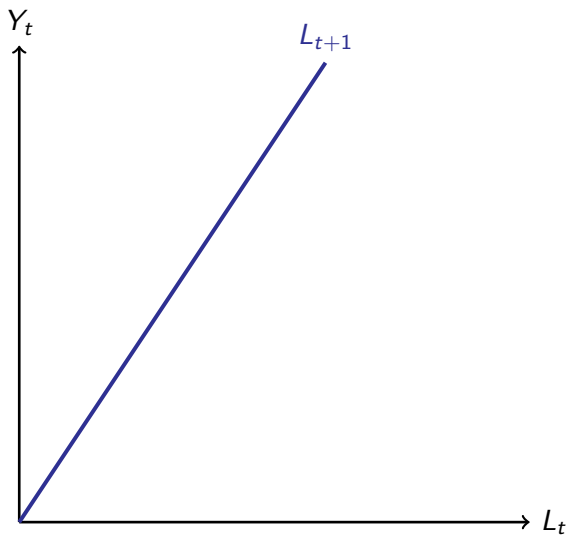
- Population size constrained by resources

*“This natural inequality of the two powers, of population, and of production in the earth, and that great law of our nature which must constantly keep their efforts equal, form the great difficulty that appears to me insurmountable in the way to the perfectibility of society... The checks which repress the superior power of population, and keep its effects on a level with the means of subsistence, are all resolvable into moral restraint, vice and misery... this constantly subsisting cause of periodical misery has existed ever since we have had any histories of mankind, does exist at present, and will for ever continue to exist, unless some decided change takes place in the physical constitution of our nature.”*

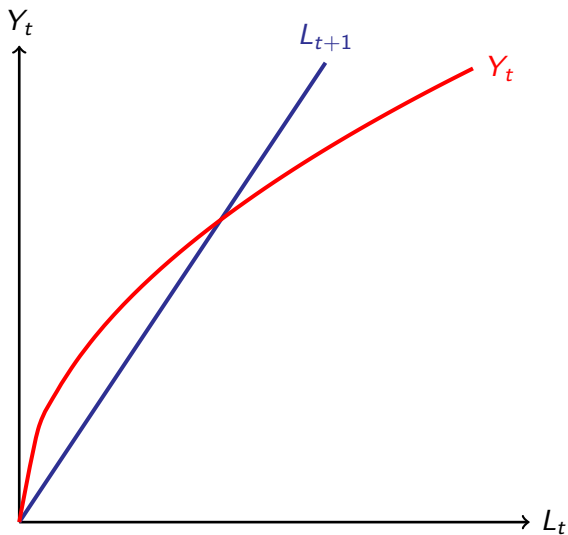
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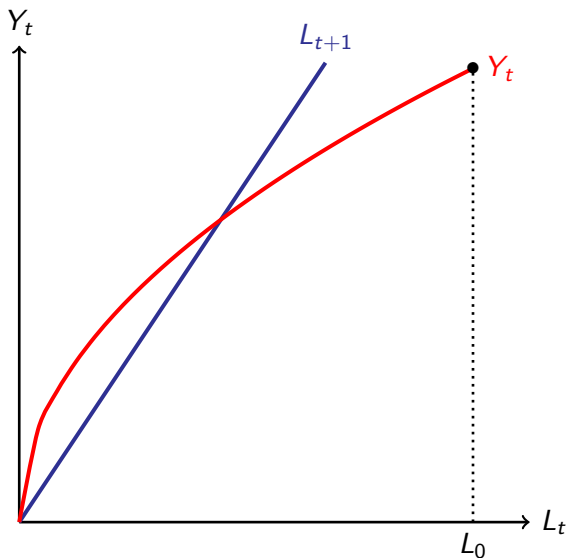
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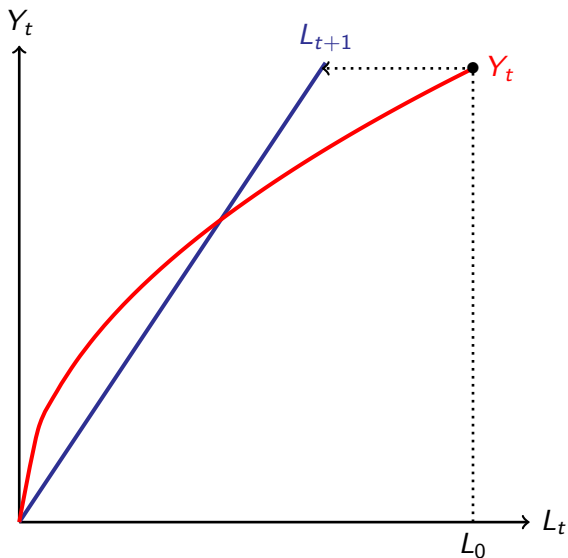
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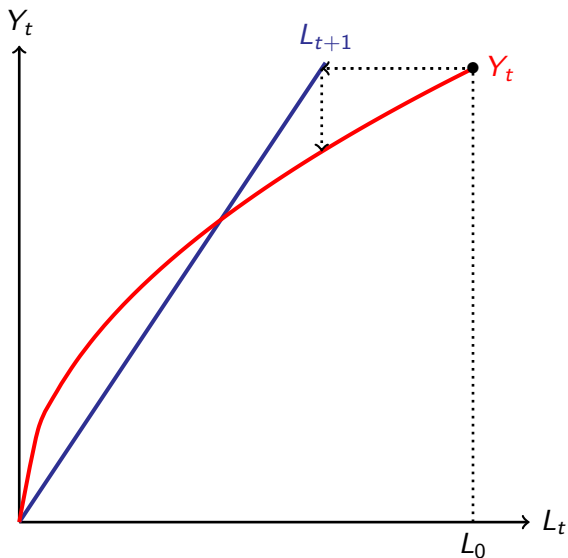
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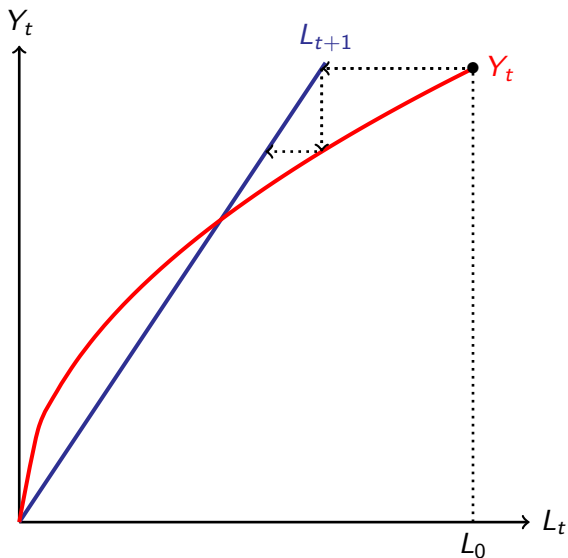
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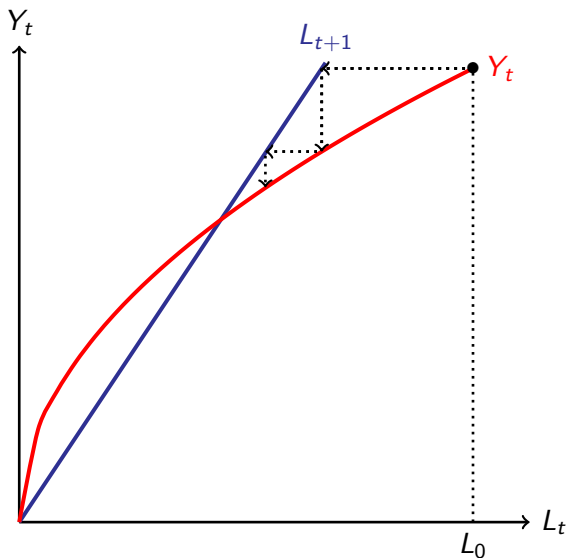


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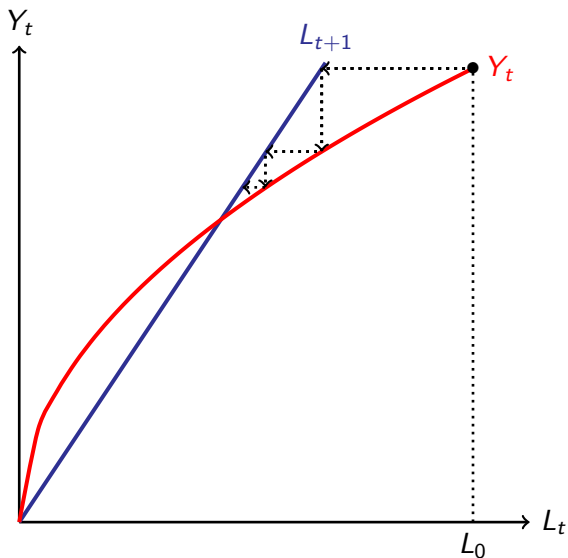




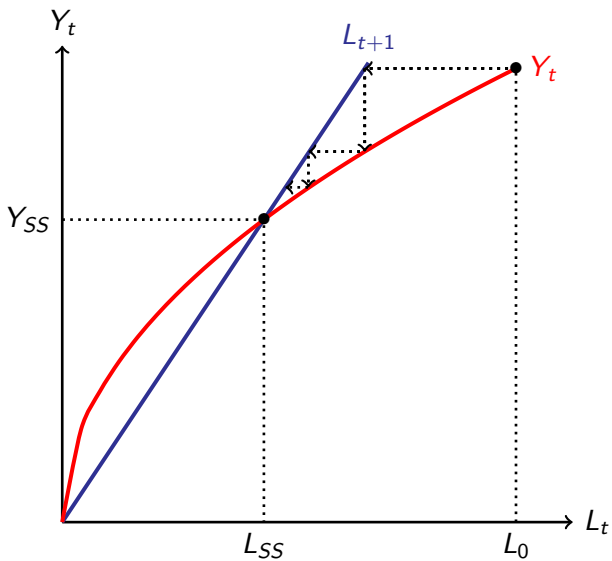
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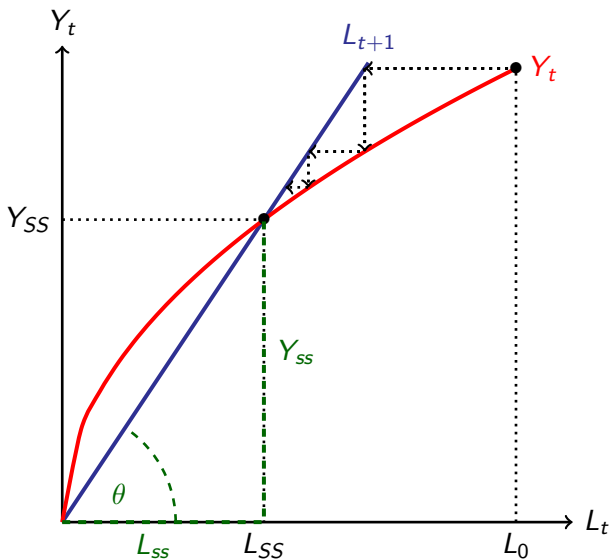
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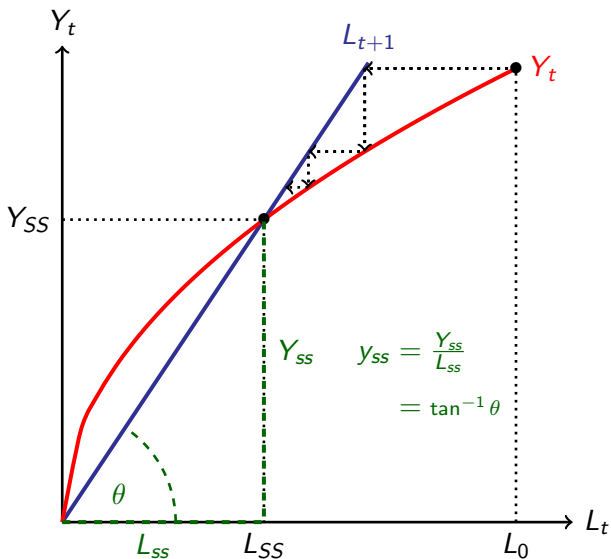
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## Malthus' Theory

- Dynamics for Living Creatures

*“Among plants and animals the view of the subject is simple. They are all impelled by a powerful instinct to the increase of their species; and this instinct is interrupted by no reasoning, or doubts about providing for their offspring. Where ever therefore there is liberty, the power of increase is exerted; and the superabundant effects are repressed afterwards by want of room and nourishment, which is common to animals and plants; and among animals, by becoming prey of others”.*

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  - abundance of seeds ( $> 10$  tons per hectare)
  - rat population explosion

## Rat Attack!

Let's see this phenomenon unfold

- Mautam
- Mizoram
- How it spreads
- Bamboo explosion
- Black Rats
- Malthusian Cycle
- Rat Explosion
- Mautam Impacts

## Malthus' Theory

- Positive checks on population size

*“are extremely various, and include every cause ... which in any degree contributes to shorten the natural duration of human life. Under this head, therefore, may be enumerated all unwholesome occupations, severe labour and exposure to the seasons, extreme poverty, bad nursing of children, great towns, excesses of all kinds, the whole train of common diseases and epidemics, wars, plague, and famine.”*



## Malthus' Theory

- Preventive checks on population size

*“Impelled to the increase of his species by an equally powerful instinct, reason interrupts his career, and asks him whether he may or not bring beings into the world, for whom he cannot provide the means of subsistence. . . . Will he not lower his rank in life? Will he not subject himself to greater difficulties than he at present feels? Will he not be obliged to labour harder? And if he has a large family, will his utmost exertions enable him to support them? May he not see his offspring in rags and misery, and clamoring for bread that he cannot give them?”*

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**4.3% population growth per year!**

## Human population growth without checks

Population	Crude Birth Rate
Hutterites (1948)	45.9
Algeria (Moslems) (1949)	34.1
Jamaica (1948)	30.9
Israel (1948)	26.8
United States (1948)	24.2

Crude Birth Rate: births/1000 people

## Human population growth without checks

Population	Fertility Ratios
Hutterites (1948)	96.3
Algeria (Moslems) (1949)	63.
Jamaica (1948)	49.
Israel (1948)	45.8
United States (1948)	42.3

Fertility Ratio: Children under 5 years of age, per 100 females 15 to 49.



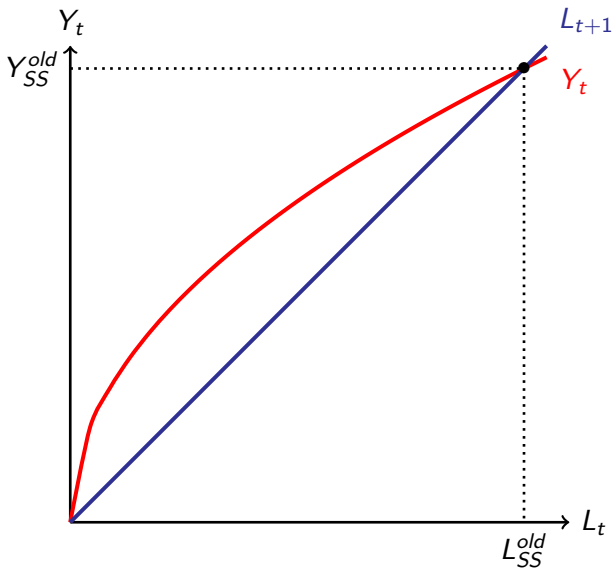
## Population Growth in Humans and Chimpanzees

Population	Type	Total Fertility Rates
<i>Humans</i>		
Hutterites	Agriculturalists	12.4
Ache	Hunter-Gatherer	8.2
Agta	Hunter-Gatherer	6.9
Hadza	Hunter-Gatherer	6.2
Hiwi	Hunter-Gatherer	5.5
Ju/'hoansi	Hunter-Gatherer	4.3
Aborigine	(Acculturated) Hunter-Gatherer	4.3
Gainj	Foragers-Horticulturalists	4.3
Tsimane	Foragers-Horticulturalists	9.2
Yanömamö	Foragers-Horticulturalists	7.9
Herero	Pastoralists	3.3
<i>Wild chimpanzees</i>		
Gombe	East Africa	6.4
Kanyawara	East Africa	7.9
Mahale	East Africa	6.9
Ngogo	East Africa	7.9
Tai	West Africa	7.5

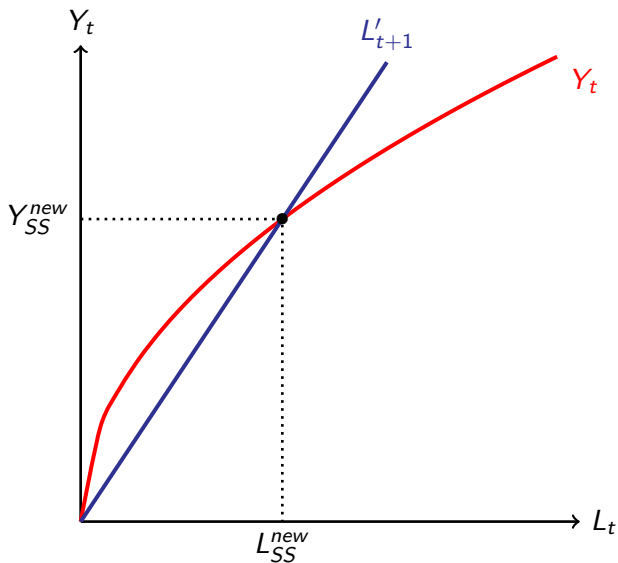
## Human population growth without checks

Initial Population	Growth Rate	Population 1000 Years later	Years to reach 10 billion
1000	0.04	$1e + 19$	411
1000	0.01	20,959,155	1,620
1000	0.001	2717	16,126
1000	0.0001	1105	161,189

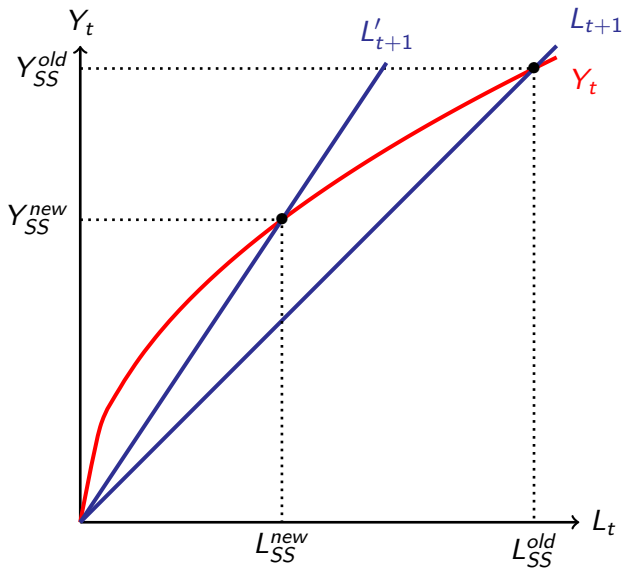
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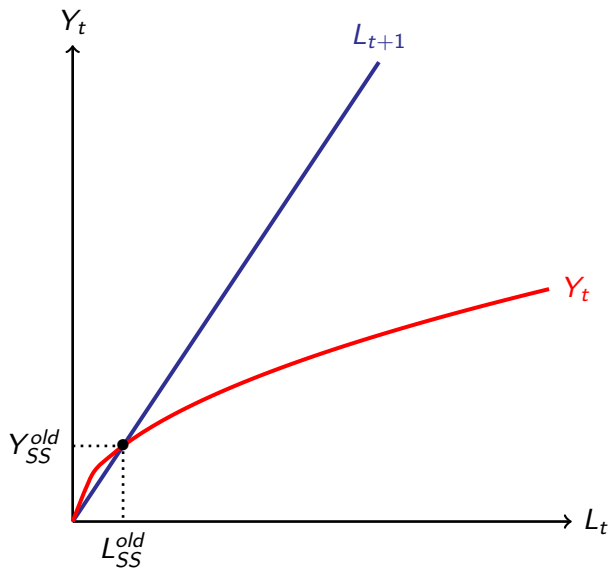
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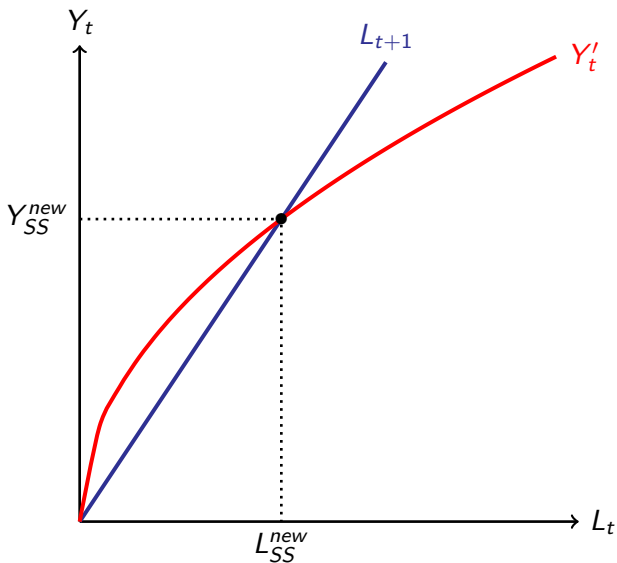
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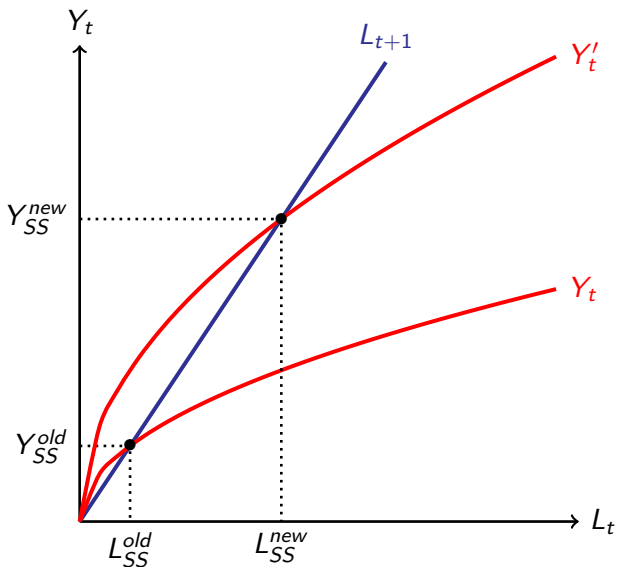
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    - (1M Famine death & 1M emigration to the New World)

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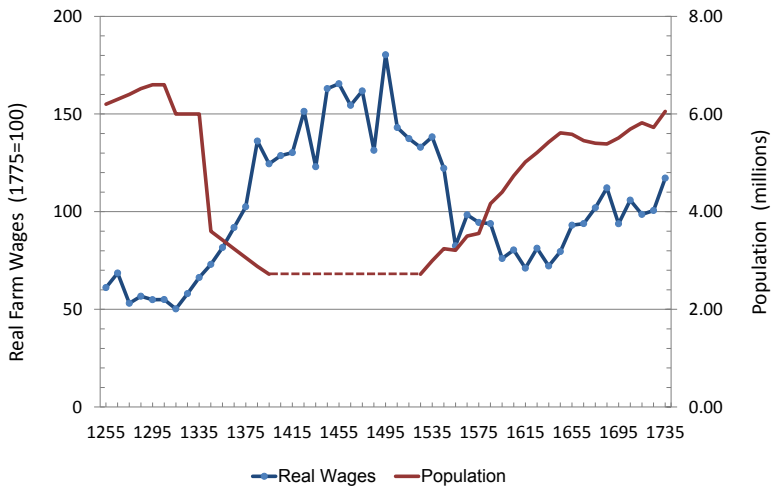
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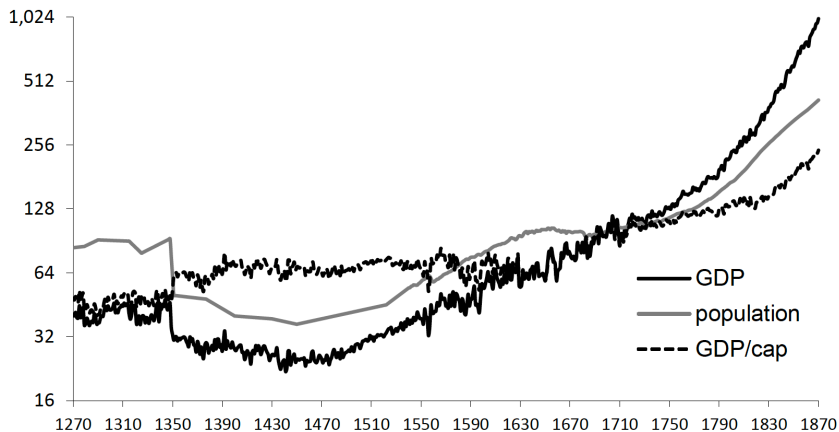
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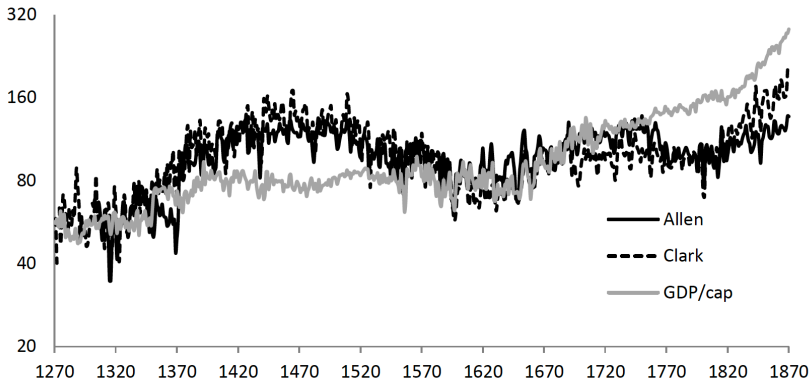
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  - ⇒ land constraints always binding

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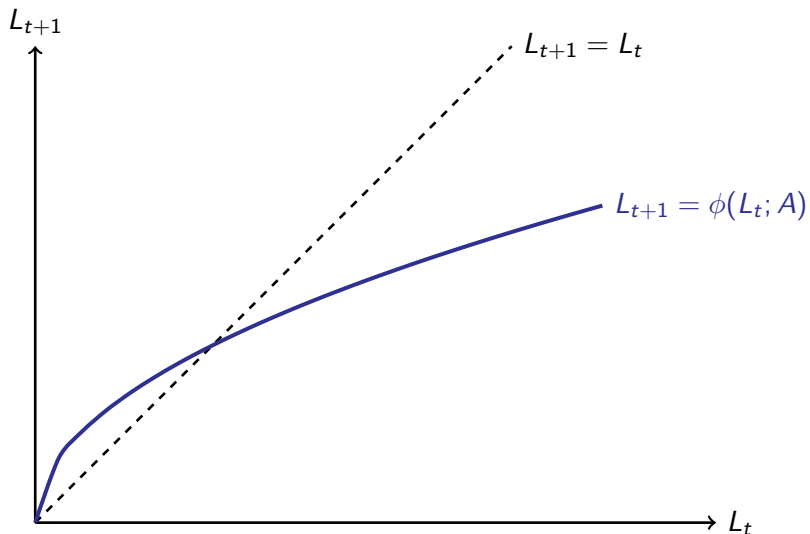
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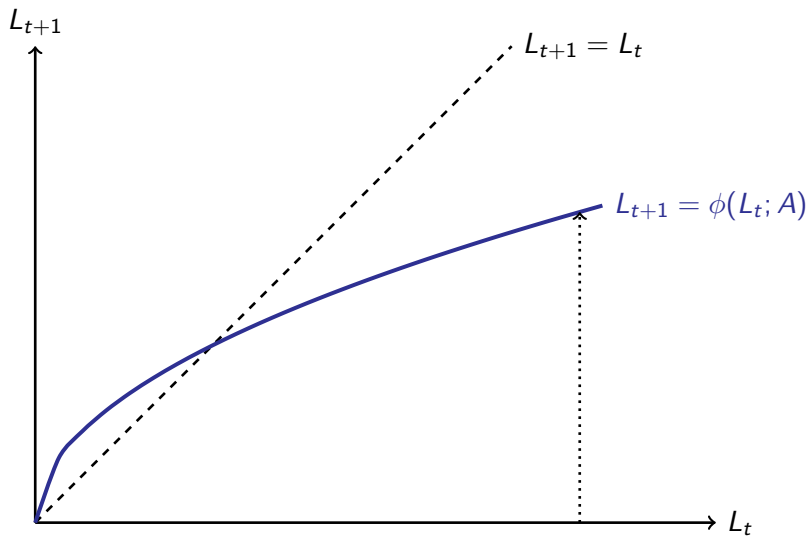
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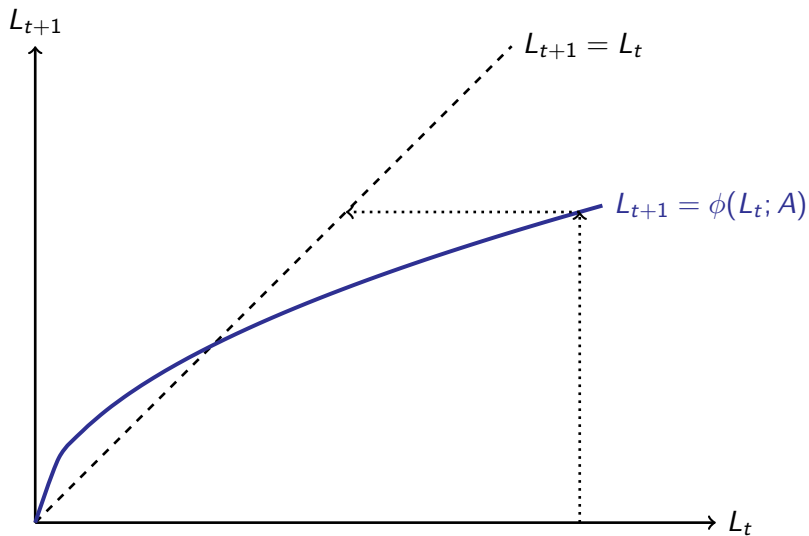


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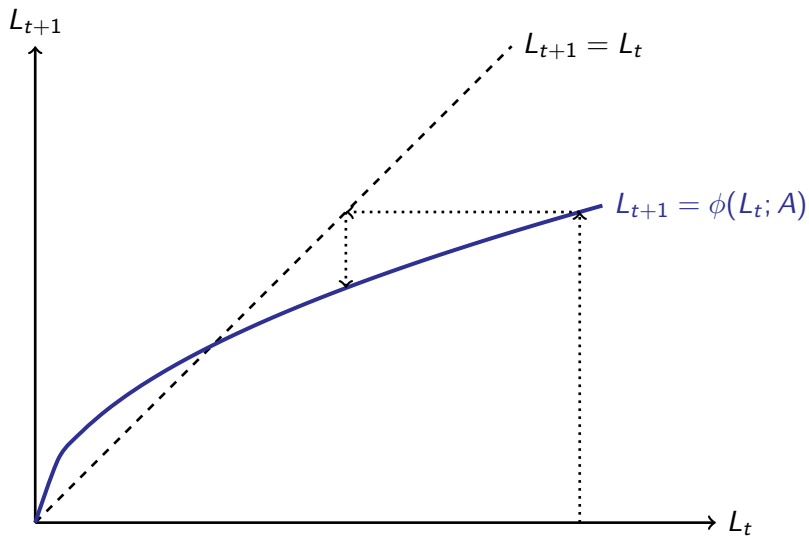




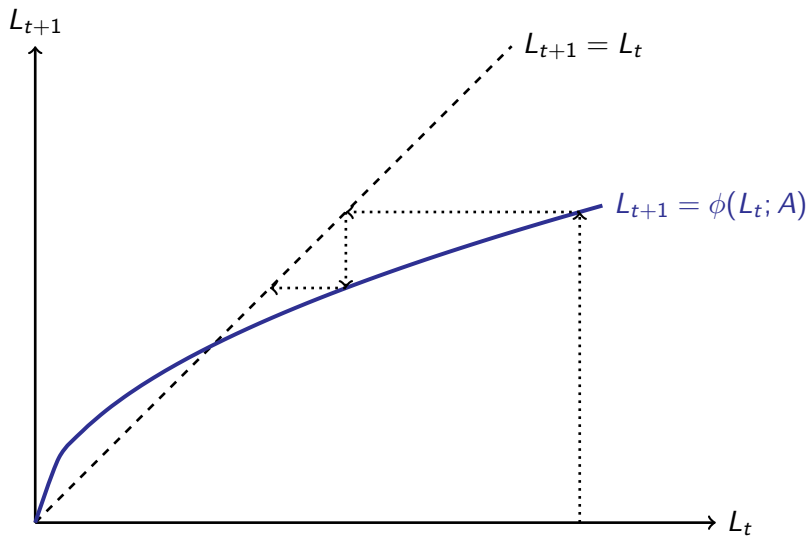
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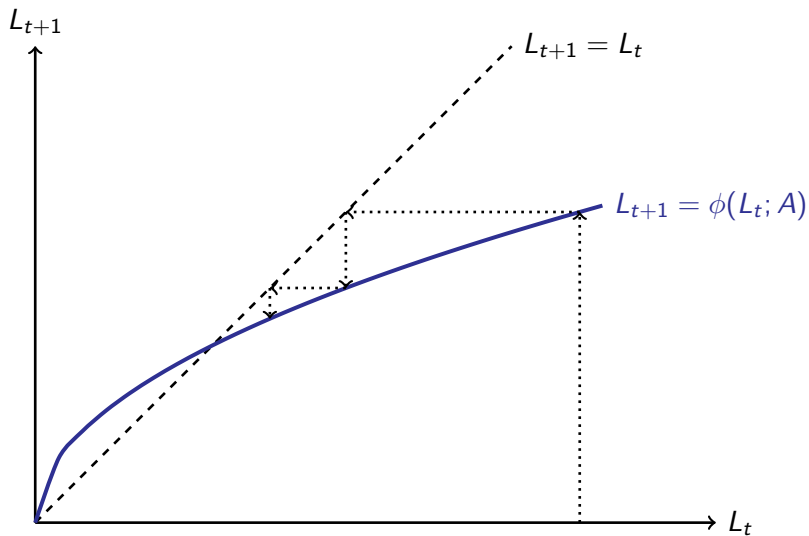
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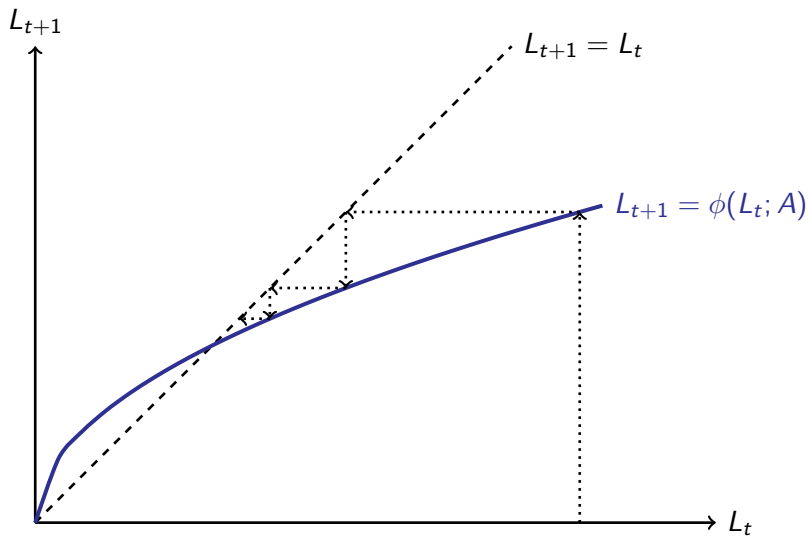
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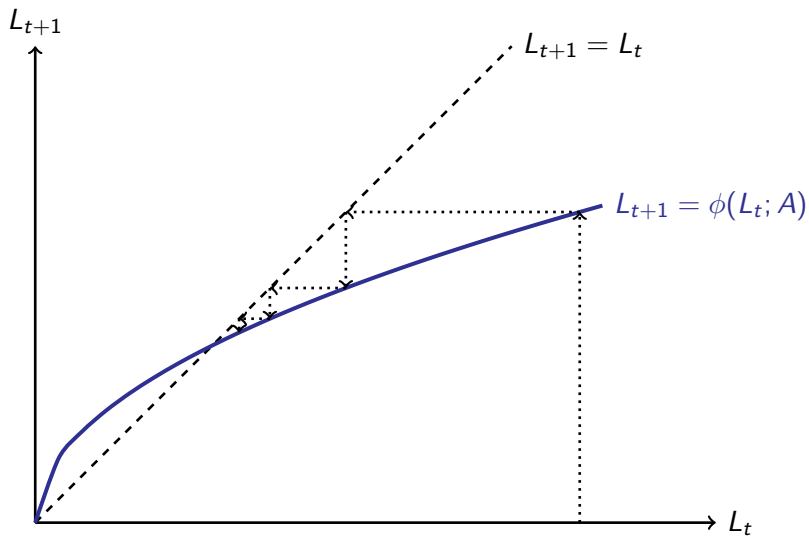
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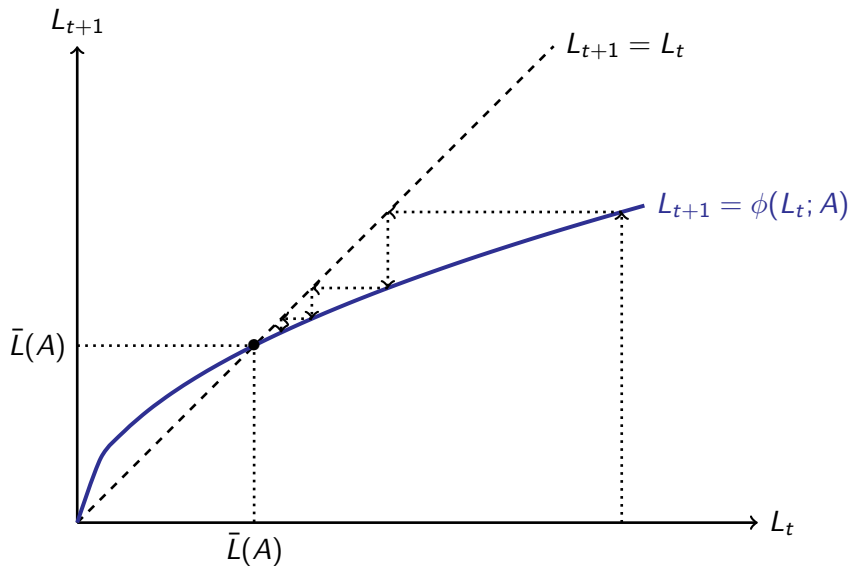
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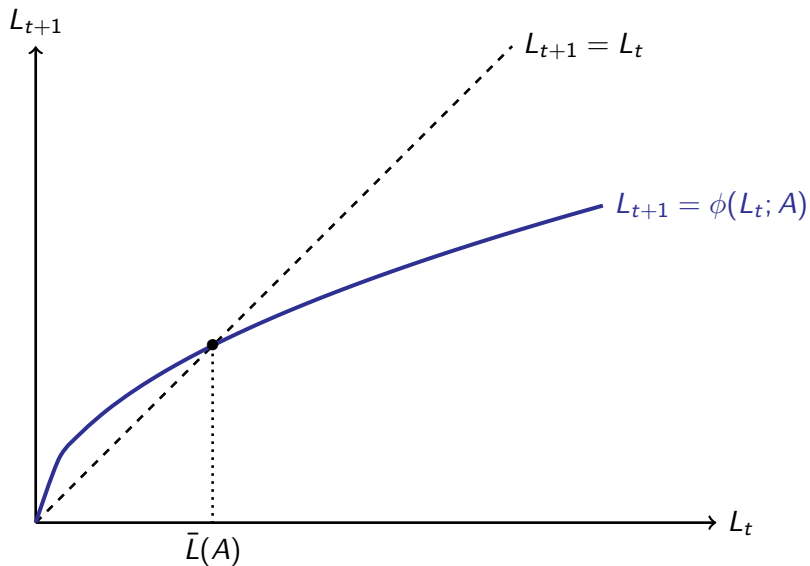
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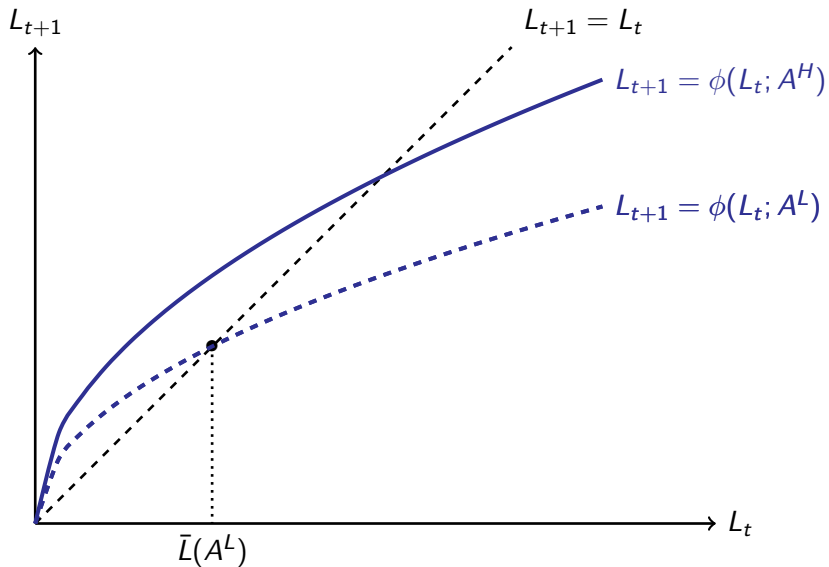
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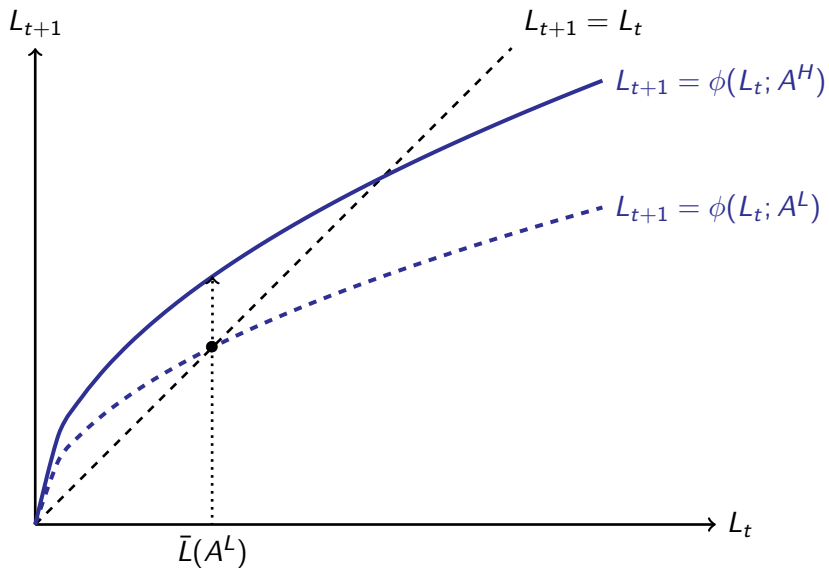




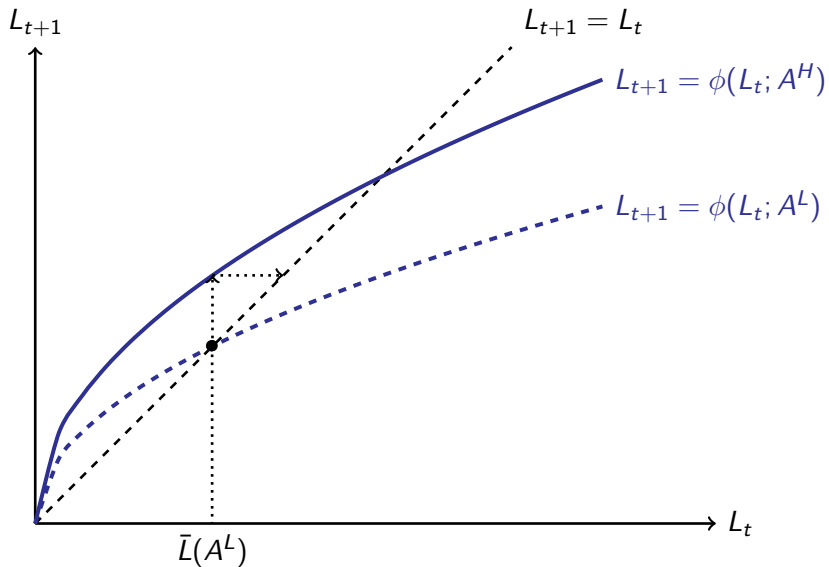
## Adjustment of Population to Advancements in Technology



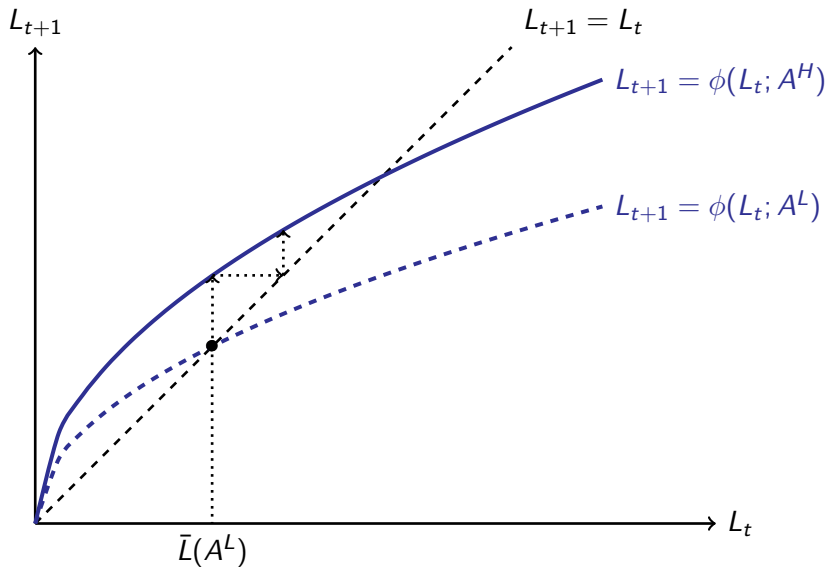
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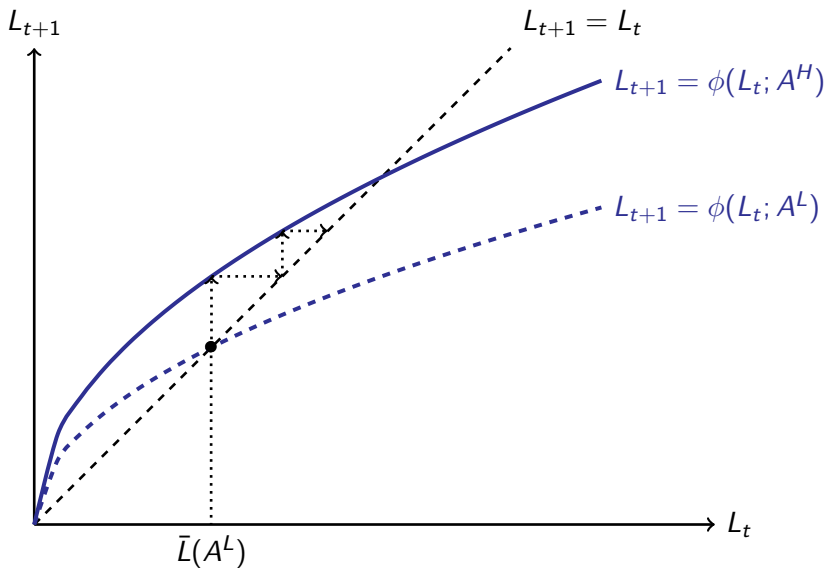
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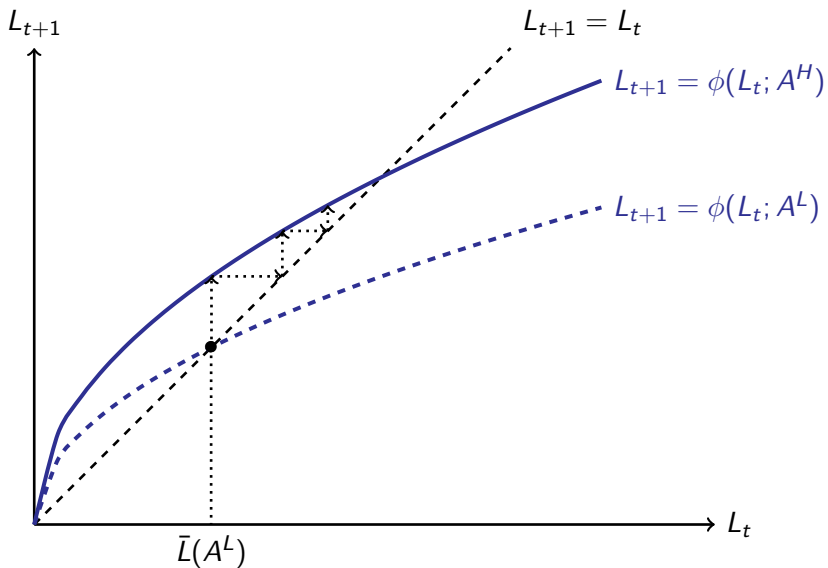
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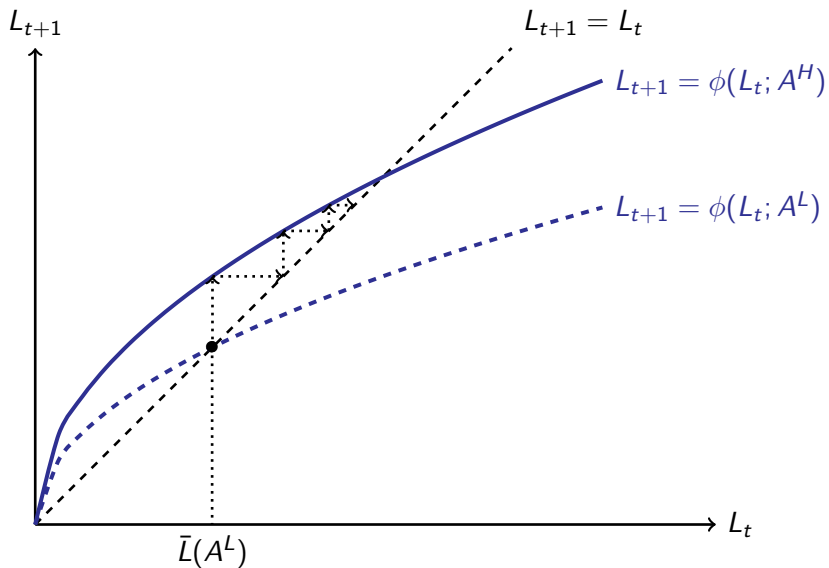
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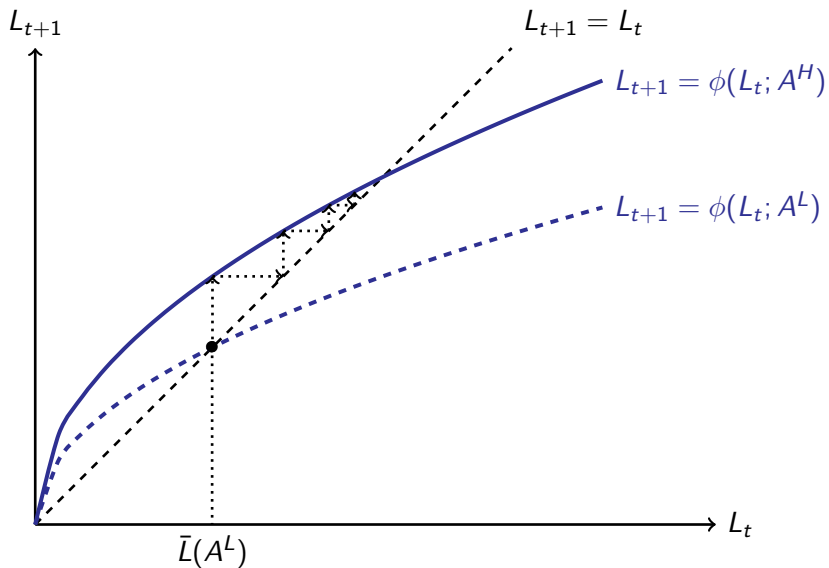
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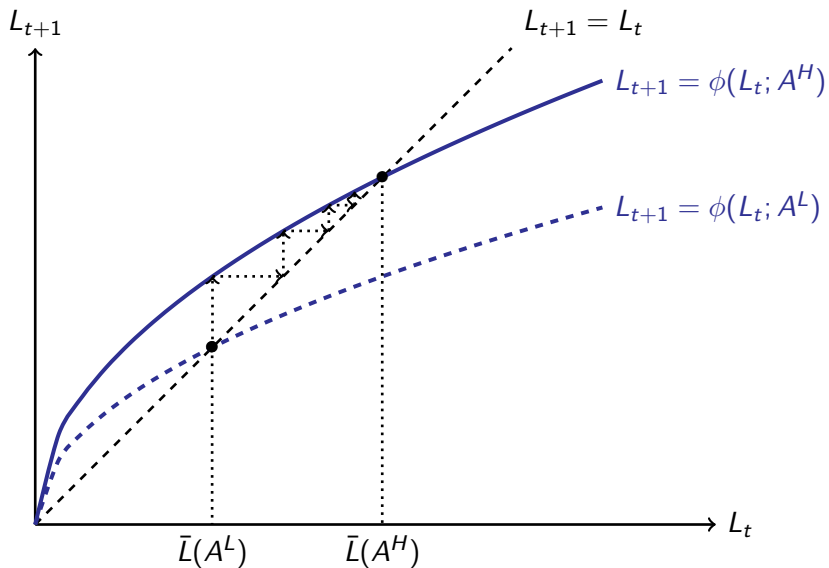


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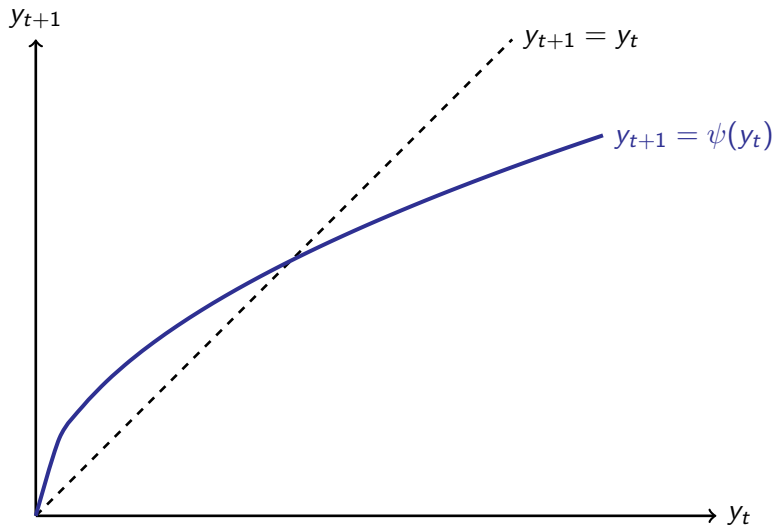
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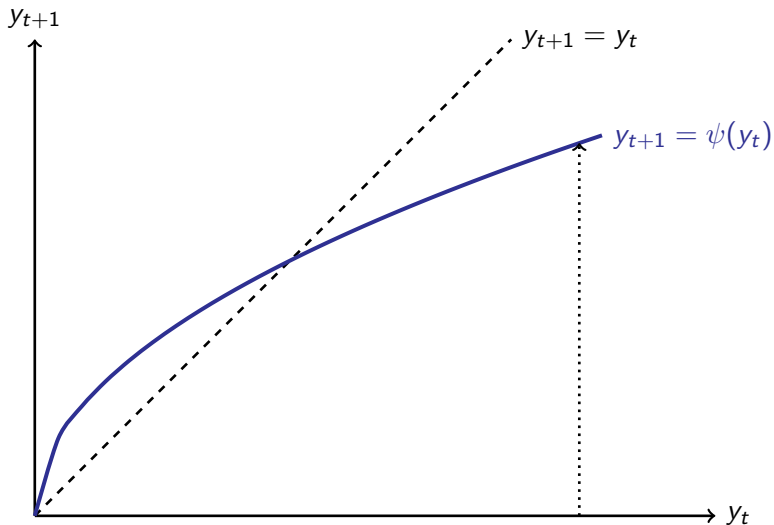
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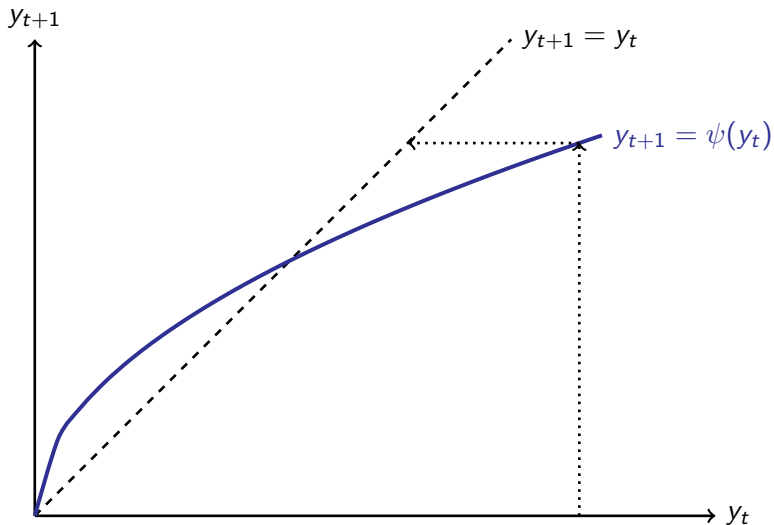




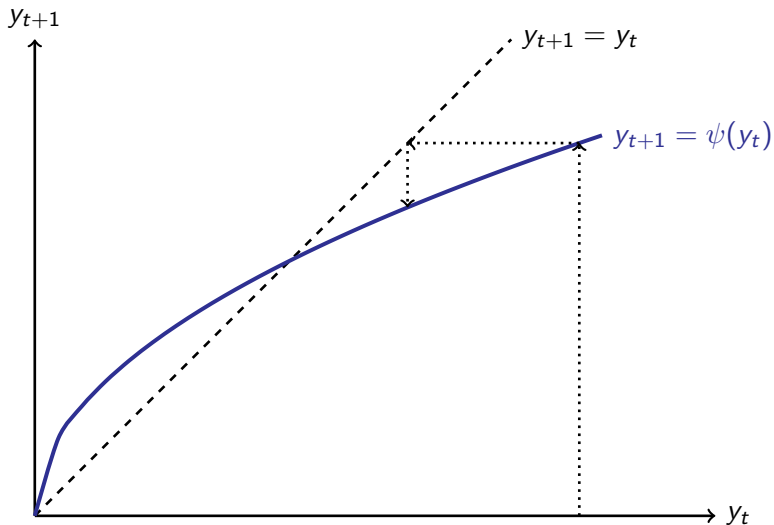
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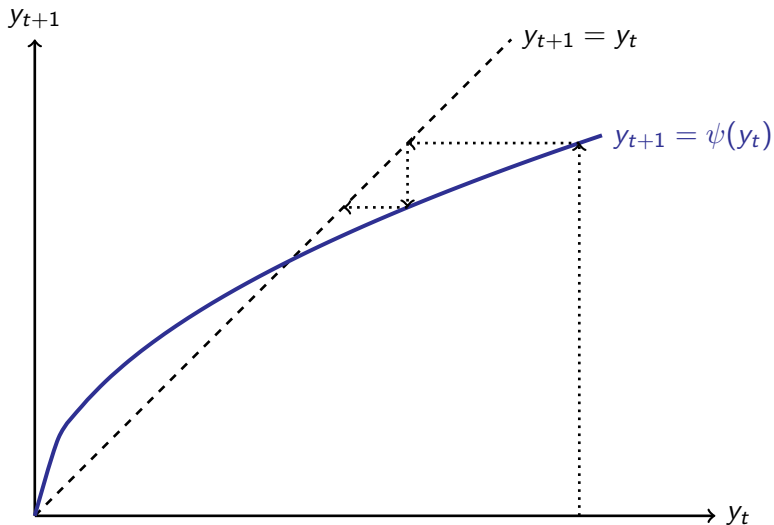
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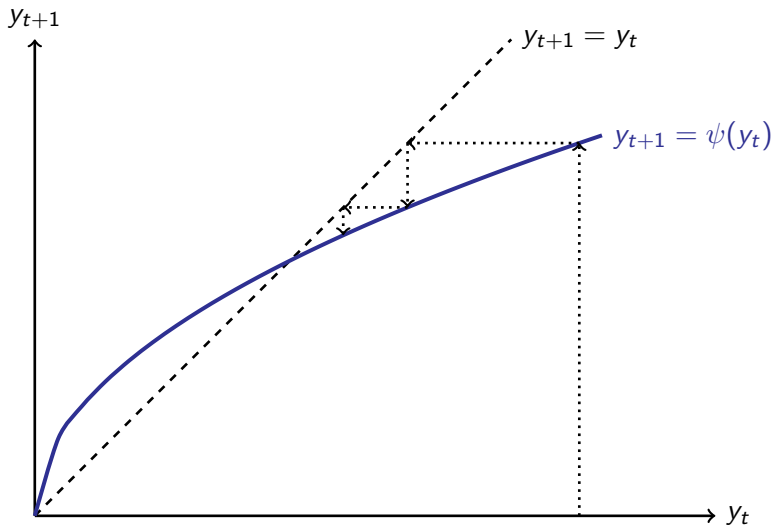
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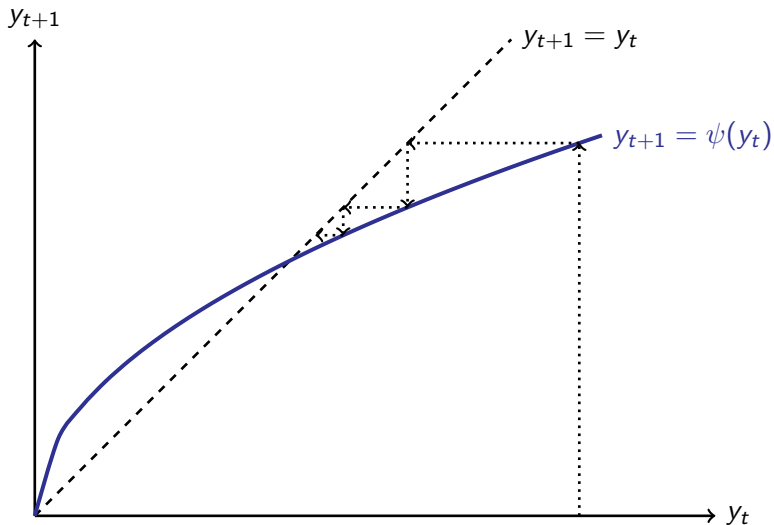
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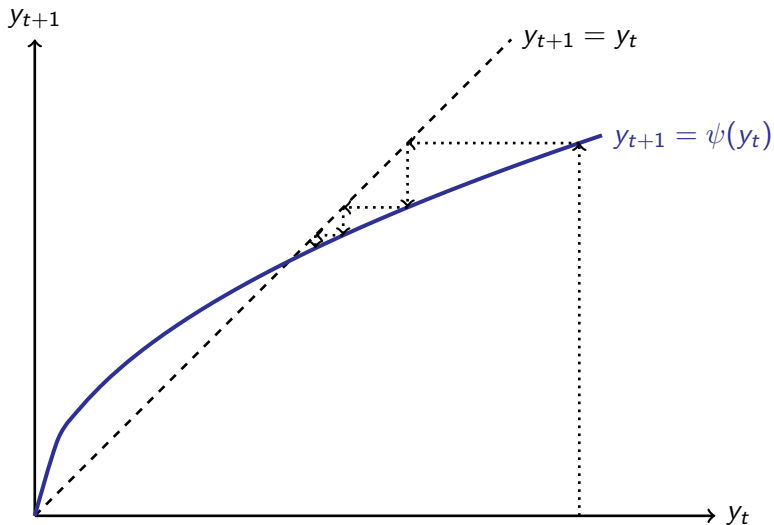
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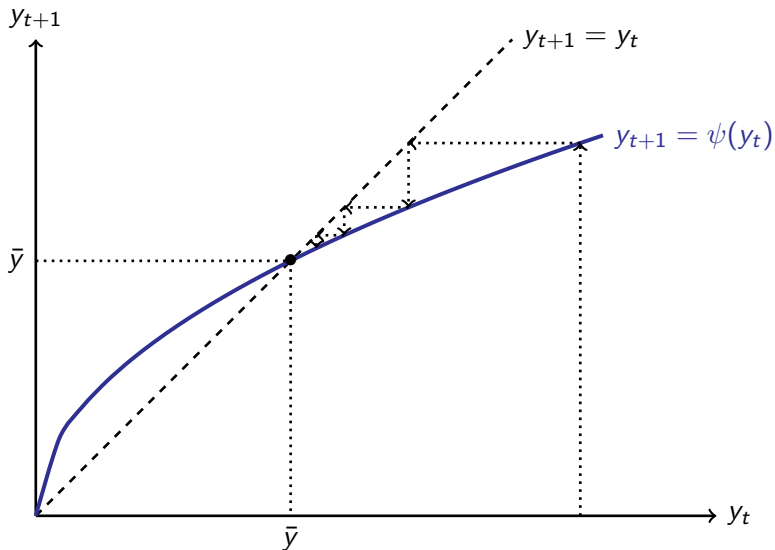
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## The Steady-State Level of Income per Worker

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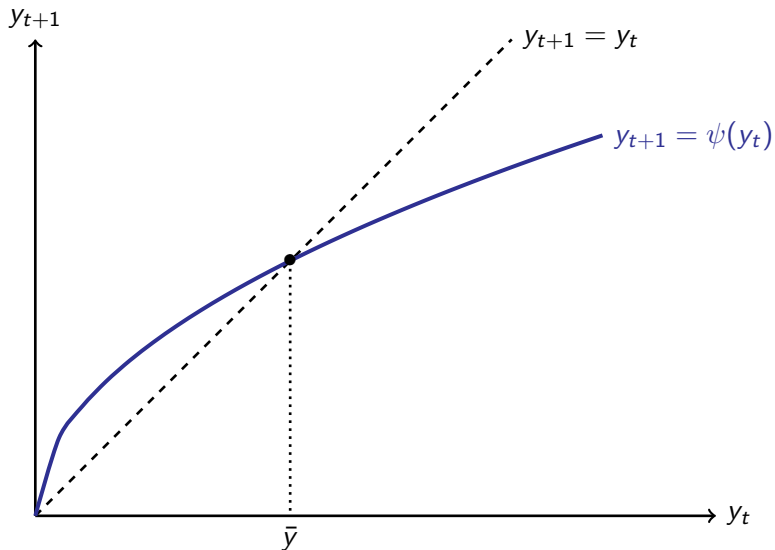
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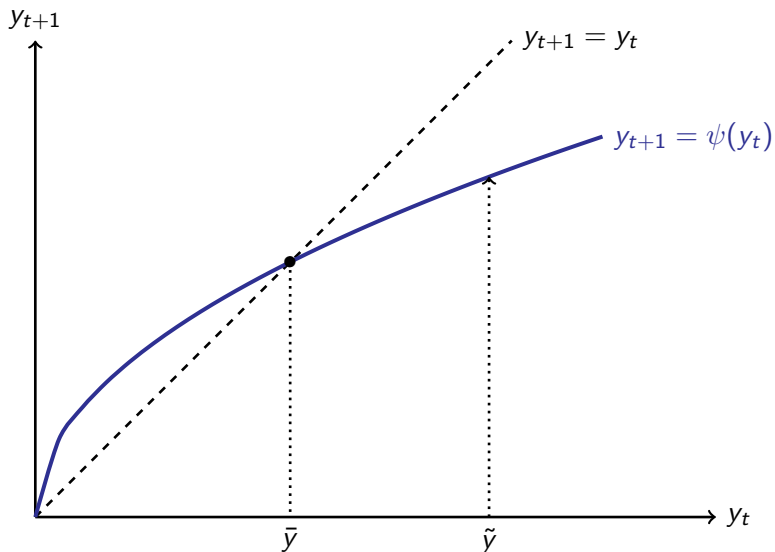
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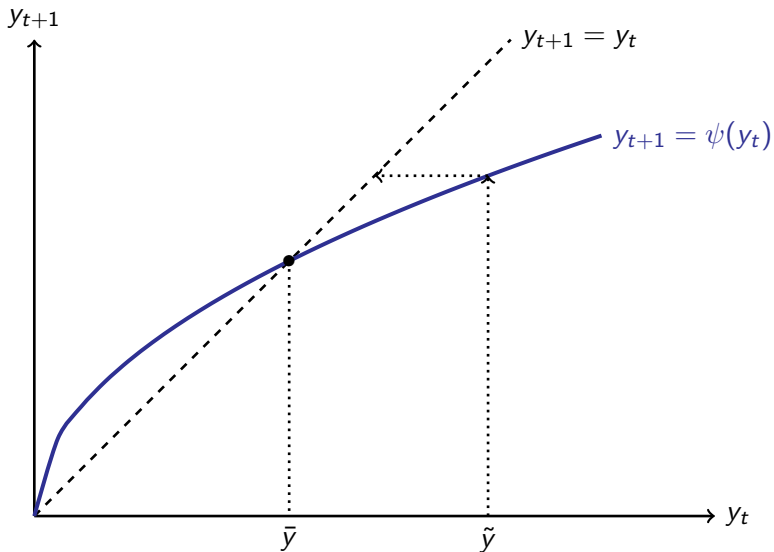
## The Evolution of Income per Worker



## The Effect of Technological Advancement on income per Worker

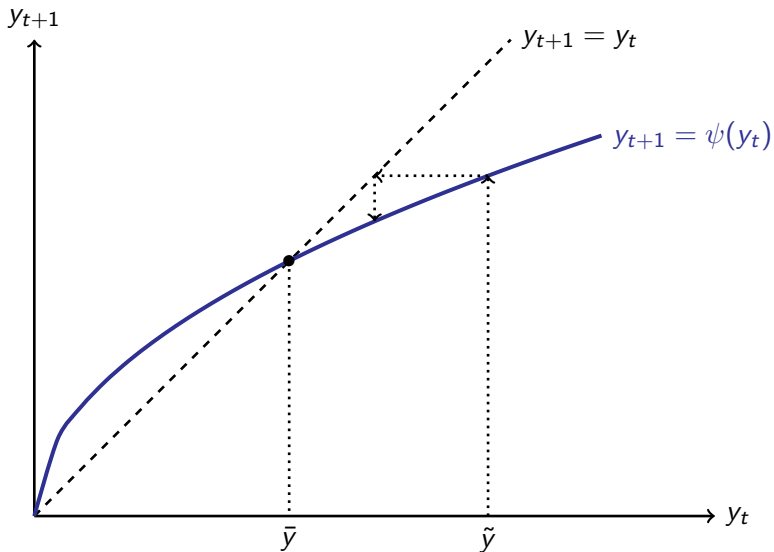


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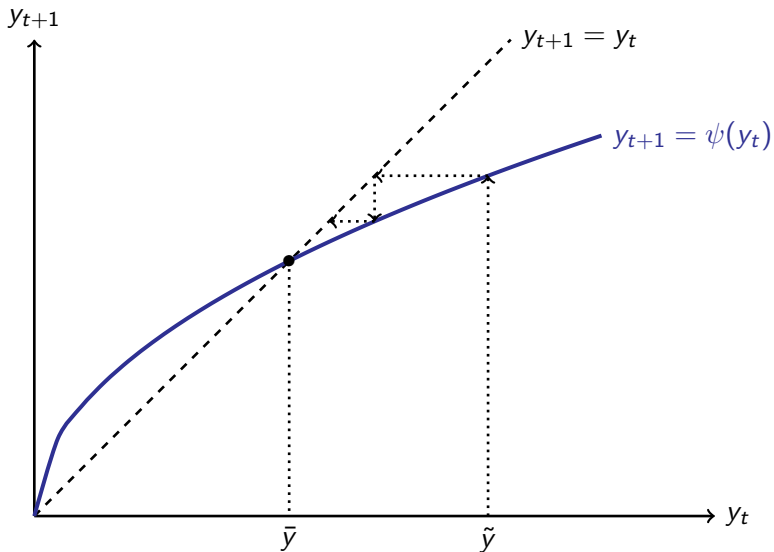




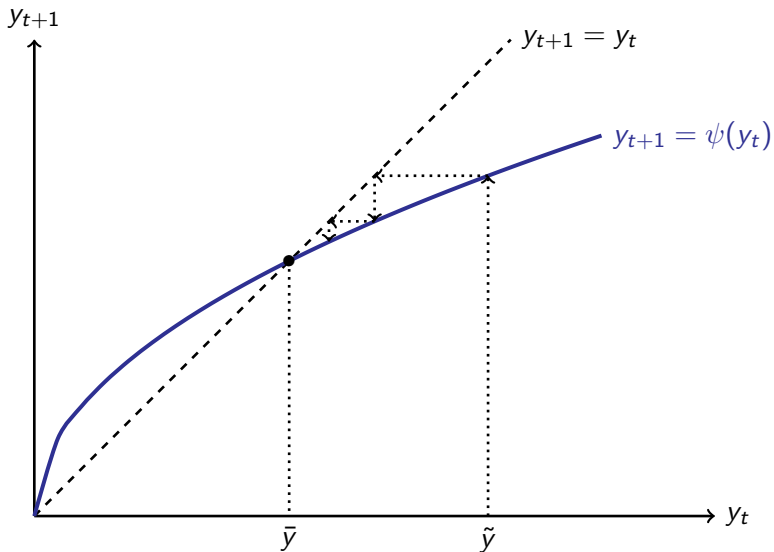
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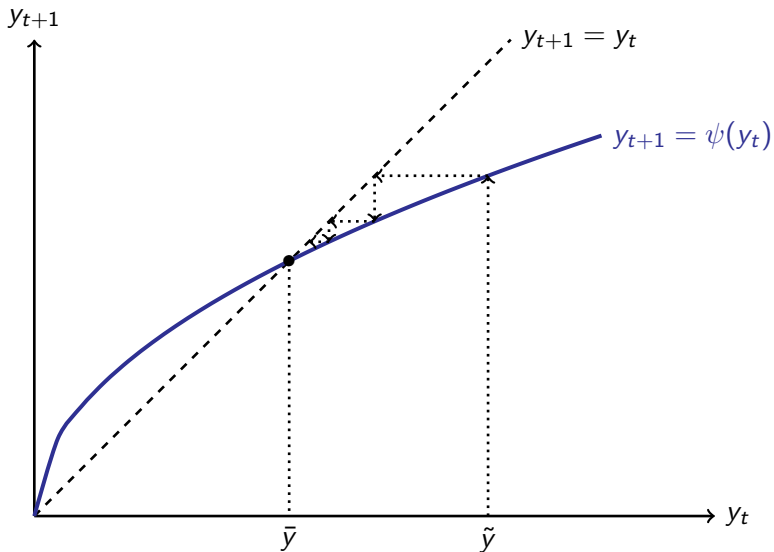
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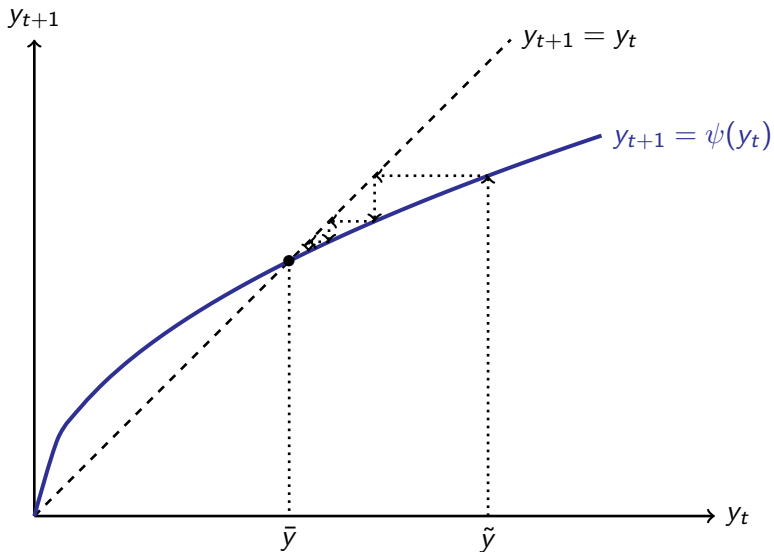
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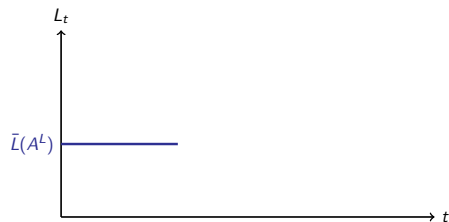
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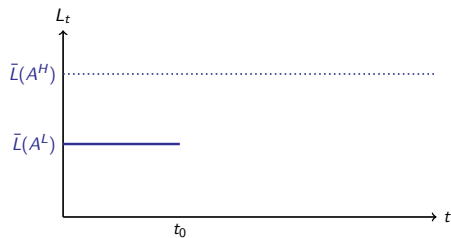
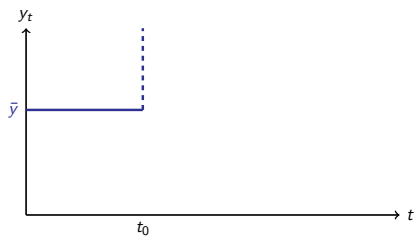
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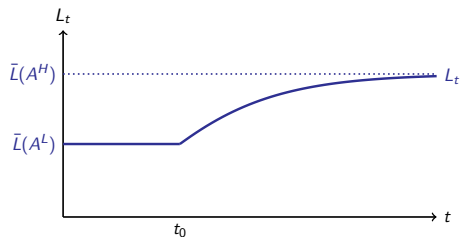
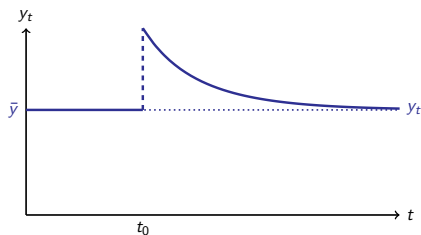
# The Effect of Technological Advancement on the Time Path of Population and Income per Worker



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## The Effect of Advancement in Technology or Land Productivity

- Increases the short-run and the steady-state level of the working population

$$\frac{\partial L_t}{\partial A} > 0 \quad \text{and} \quad \frac{\partial \bar{L}}{\partial A} > 0$$

- Increases the level of income per capita in the short-run but does not affect the steady-state levels of income per worker

$$\frac{\partial y_t}{\partial A} > 0 \quad \text{and} \quad \frac{\partial \bar{y}}{\partial A} = 0$$

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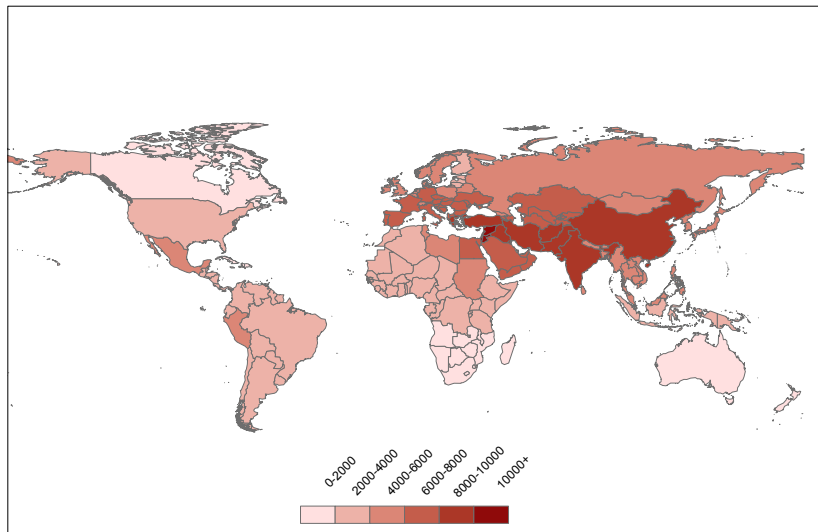
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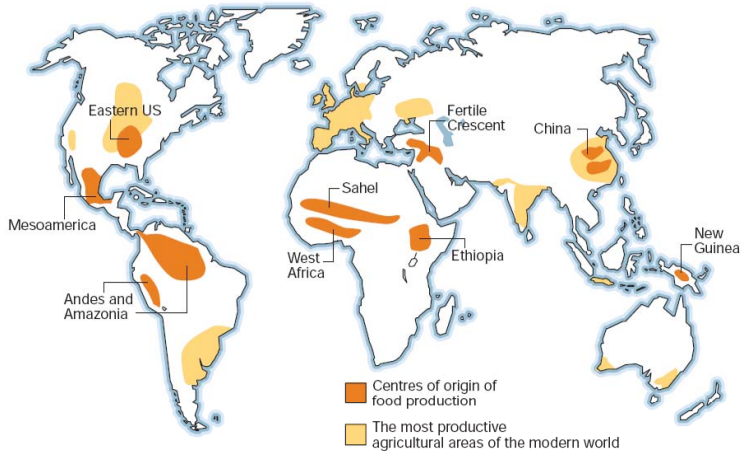
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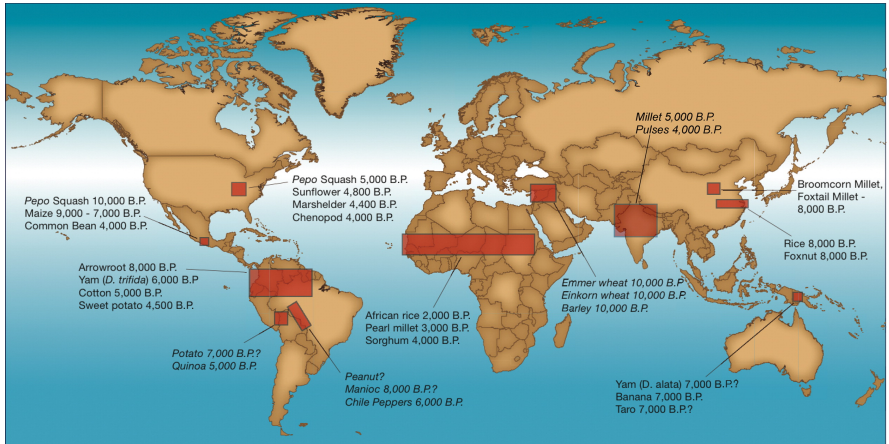
## Variation in the Onset of the Neolithic Revolution



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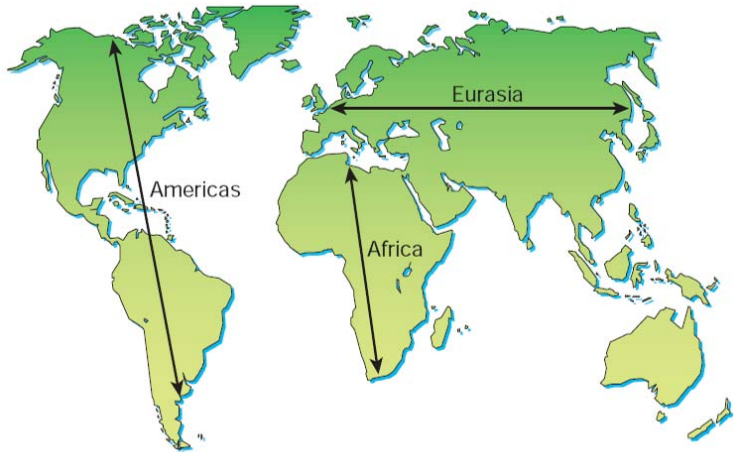
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## Orientation of Continents



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## The Neolithic Revolution &amp; Technological Level: 1000 BCE–1500 CE

	Technology Level 1000BCE-1500CE					
	1000BCE		1CE		1500CE	
	(1)	(2)	(3)	(4)	(5)	(6)
Years Since Neolithic Revolution	0.72*** (0.06)	0.47*** (0.12)	0.56*** (0.06)	0.28** (0.12)	0.74*** (0.06)	0.34*** (0.10)
Continental FE	No	Yes	No	Yes	No	Yes
Additional Geographical Controls	No	Yes	No	Yes	No	Yes
Adjusted- $R^2$	0.51	0.60	0.31	0.63	0.55	0.82
Observations	112	111	134	133	113	112

Notes: Standardized coefficients from an Ordinary Least Squares (OLS) regression. Heteroskedasticity robust standard error estimates are reported in parentheses; \*\*\* denotes statistical significance at the 1% level, \*\* at the 5% level, and \* at the 10% level, all for two-sided hypothesis tests.

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$$\ln P_{i,t} = \alpha_{0,t} + \alpha_{1,t} \ln T_{i,t} + \alpha_{2,t} \ln X_i + \alpha'_{3,t} \Gamma_i + \alpha'_{4,t} D_i + \delta_{i,t}$$

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$$\ln y_{i,t} = \beta_{0,t} + \beta_{1,t} \ln T_{i,t} + \beta_{2,t} \ln X_i + \beta'_{3,t} \Gamma_i + \beta'_{4,t} D_i + \varepsilon_{i,t}$$

- $P_{i,t} \equiv$  population density of country  $i$  in year  $t$
- $y_{i,t} \equiv$  income per capita of country  $i$  in year  $t$
- $T_i \equiv$  years elapsed since the onset of agriculture in country  $i$
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- $D_i \equiv$  vector of continental fixed effect in country  $i$

## Determinants of Population Density in 1500 CE

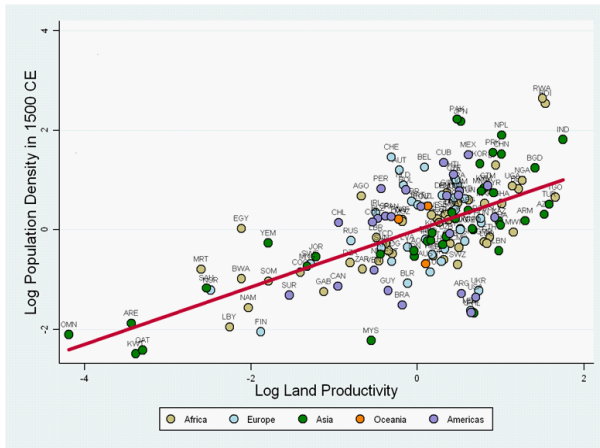
	(1)	(2)	(3)	(4)	(5)	(6)
	OLS	OLS	OLS	OLS	OLS	IV
<b>Dependent Variable: Log population density in 1500 CE</b>						
Log years since Neolithic	<b>0.833***</b> (0.298)		<b>1.025***</b> (0.223)	<b>1.087***</b> (0.184)	<b>1.389***</b> (0.224)	<b>2.077***</b> (0.391)
Log land productivity		<b>0.587***</b> (0.071)	<b>0.641***</b> (0.059)	<b>0.576***</b> (0.052)	<b>0.573***</b> (0.095)	<b>0.571***</b> (0.082)
Log absolute latitude		-0.425*** (0.124)	-0.353*** (0.104)	-0.314*** (0.103)	-0.278** (0.131)	-0.248** (0.117)
Distance to nearest coast or river				-0.392*** (0.142)	0.220 (0.346)	0.250 (0.333)
% land within 100 km of coast or river				0.899*** (0.282)	1.185*** (0.377)	1.350*** (0.380)
Continental dummies	Yes	Yes	Yes	Yes	Yes	Yes
Observations	147	147	147	147	96	96
R <sup>2</sup>	0.40	0.60	0.66	0.73	0.73	0.70
First-stage F-statistic						14.65
Overident. p-value						0.44
Notes: Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1						

## Effects on Income per Capita versus Population Density

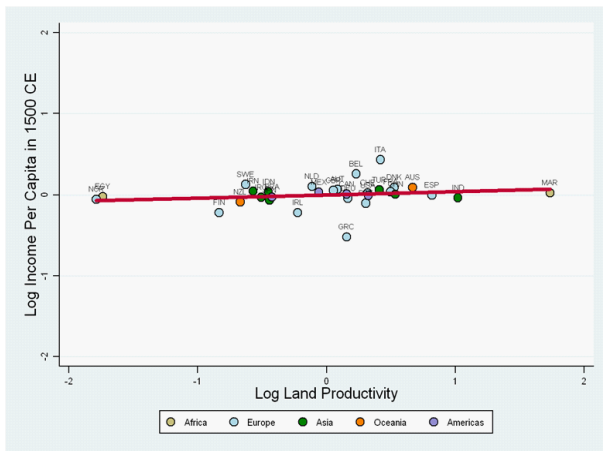
	OLS	OLS	OLS	OLS	OLS	OLS
	(1)	(2)	(3)	(4)	(5)	(6)
	Log Income Per Capita in			Log Population Density in		
	1500 CE	1000 CE	1 CE	1500 CE	1000 CE	1 CE
Log years since Neolithic	<b>0.159</b> (0.136)	<b>0.073</b> (0.045)	<b>0.109</b> (0.072)	<b>1.337**</b> (0.594)	<b>0.832**</b> (0.363)	<b>1.006**</b> (0.483)
Log land productivity	<b>0.041</b> (0.025)	<b>-0.021</b> (0.025)	<b>-0.001</b> (0.027)	<b>0.584***</b> (0.159)	<b>0.364***</b> (0.110)	<b>0.681**</b> (0.255)
Log absolute latitude	-0.041 (0.073)	0.060 (0.147)	-0.175 (0.175)	0.050 (0.463)	-2.140** (0.801)	-2.163** (0.979)
Distance to nearest coast or river	0.215 (0.198)	-0.111 (0.138)	0.043 (0.159)	-0.429 (1.237)	-0.237 (0.751)	0.118 (0.883)
% land within 100 km of coast or river	0.124 (0.145)	-0.150 (0.121)	0.042 (0.127)	1.855** (0.820)	1.326** (0.615)	0.228 (0.919)
Continental dummies	Yes	Yes	Yes	Yes	Yes	Yes
Observations	31	26	29	31	26	29
R <sup>2</sup>	0.66	0.68	0.33	0.88	0.95	0.89

Notes: Robust standard errors in parentheses; \*\*\* p<0.01, \*\* p<0.05, \* p<0.1

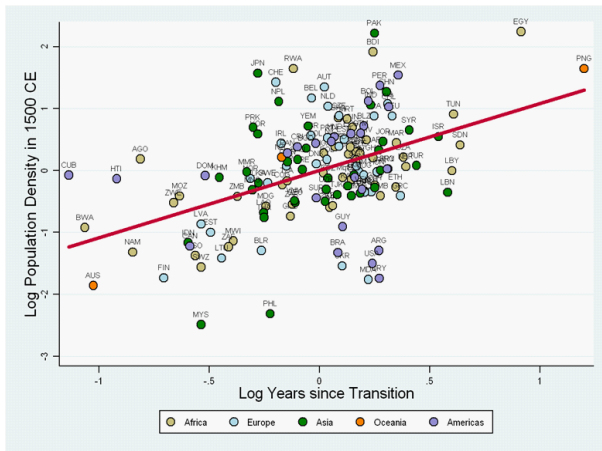
# Land Productivity and Population Density in 1500



# Land Productivity and Income per Capita in 1500

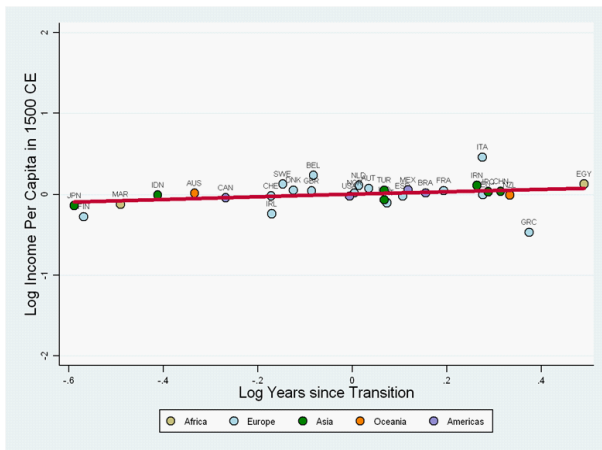


# Technology and Population Density in 1500





# Technology and Income per Capita in 1500



## Robustness of Identification Strategy

- Robustness to the inclusion of direct measures of technology

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  - Exploit variation in a direct measure of the technology level

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- Robustness to the exclusion of unobserved time-invariant country fixed effects
  - First-difference estimation strategy (with a lagged explanatory variable)
  - The effect of changes in the level of technology in 1000 BCE-1 CE on population density and income per capita in 1-1000CE



## Robustness to Direct Measures of Technological Level

	OLS (1)	OLS (2)	OLS (3)	OLS (4)	OLS (5)	OLS (6)
	Dependent Variable:					
	Log Population Density in:		Log Income Per Capita in:		Log Population Density in:	
	1000 CE	1 CE	1000 CE	1 CE	1000 CE	1 CE
Log Technology Index in Relevant Period	4.315*** (0.850)	4.216*** (0.745)	0.064 (0.230)	0.678 (0.432)	12.762*** (0.918)	7.461** (3.181)
Log land productivity	0.449*** (0.056)	0.379*** (0.082)	-0.016 (0.030)	0.004 (0.033)	0.429** (0.182)	0.725** (0.303)
Log absolute latitude	-0.283** (0.120)	-0.051 (0.127)	0.036 (0.161)	-0.198 (0.176)	-1.919*** (0.576)	-2.350*** (0.784)
Distance to nearest coast or river	-0.638*** (0.188)	-0.782*** (0.198)	-0.092 (0.144)	0.114 (0.164)	0.609 (0.469)	0.886 (0.904)
% land within 100 km of coast or river	0.385 (0.313)	0.237 (0.329)	-0.156 (0.139)	0.092 (0.136)	1.265** (0.555)	0.788 (0.934)
Continental dummies	Yes	Yes	Yes	Yes	Yes	Yes
Observations	140	129	26	29	26	29
R <sup>2</sup>	0.61	0.62	0.64	0.30	0.97	0.88
Notes: Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1						

## The Causal Effect of Technological Level on Population Density

	OLS (1)	OLS (2)	IV (3)	OLS (4)	OLS (5)	IV (6)
	Dependent Variable: Population Density in: 1000CE			1CE		
Log Technology Index in Relevant Period	4.315*** (0.850)	4.198*** (1.164)	14.530*** (4.437)	4.216*** (0.745)	3.947*** (0.983)	10.798*** (2.857)
Log land productivity	0.449*** (0.056)	0.498*** (0.139)	0.572*** (0.148)	0.379*** (0.082)	0.350** (0.172)	0.464** (0.182)
Log absolute latitude	-0.283** (0.120)	-0.185 (0.151)	-0.209 (0.209)	-0.051 (0.127)	0.083 (0.170)	-0.052 (0.214)
Distance to nearest coast or river	-0.638*** (0.188)	-0.363 (0.426)	-1.155* (0.640)	-0.782*** (0.198)	-0.625 (0.434)	-0.616 (0.834)
% land within 100 km of coast or river	0.385 (0.313)	0.442 (0.422)	0.153 (0.606)	0.237 (0.329)	0.146 (0.424)	-0.172 (0.642)
Continental dummies	Yes	Yes	Yes	Yes	Yes	Yes
Observations	140	92	92	129	83	83
R <sup>2</sup>	0.61	0.55	0.13	0.62	0.58	0.32
First-stage F-statistic			12.52			12.00
Overid. p-value			0.941			0.160
Notes: Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1						

## Robustness to Technology Diffusion and other Geographic Characteristics

	(1)	(2)	(3)	(4)	(5)	(6)
	Log Population		Log Income Per		Log Population	
	Density in 1500		Capita in 1500		Density in 1500	
Log Technology Index in Relevant Period	0.828*** (0.208)	0.877*** (0.214)	0.117 (0.221)	0.103 (0.214)	1.498** (0.546)	1.478** (0.556)
Log land productivity	0.559*** (0.048)	0.545*** (0.063)	0.036 (0.032)	0.047 (0.037)	0.596*** (0.123)	0.691*** (0.122)
Log Distance to Frontier	-0.186*** (0.035)	-0.191*** (0.036)	-0.005 (0.011)	-0.001 (0.013)	-0.130* (0.066)	-0.108* (0.055)
Small Island Dummy	0.067 (0.582)	0.086 (0.626)	-0.118 (0.216)	-0.046 (0.198)	1.962** (0.709)	2.720*** (0.699)
Landlocked Dummy	0.131 (0.209)	0.119 (0.203)	0.056 (0.084)	0.024 (0.101)	1.490*** (0.293)	1.269*** (0.282)
% Land in Temperate Climate Zones		-0.196 (0.513)		-0.192 (0.180)		-1.624* (0.917)
Continental dummies	Yes	Yes	Yes	Yes	Yes	Yes
Observations	147	147	31	31	31	31
R <sup>2</sup>	0.76	0.76	0.67	0.67	0.94	0.96

## Conclusions

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### Malthusian Hypothesis

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- Income per capita levels

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  - Land Productivity
- Income per capita levels
  - Independent of both



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  - Land Productivity
- Income per capita levels
  - Independent of both
  - Determined by preferences for children