Inequality and the Process of Development

Ömer Özak

Department of Economics Southern Methodist University

Economic Growth and Comparative Development

Inequality is beneficial for growth (in the post-industrialization stage)

- The marginal propensity to save increases with income
- Inequality channels resources towards individuals whose marginal propensity to save is higher
 - ⇒ increases aggregate savings & capital accumulation
 - ⇒ enhances the development process

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 - Growth Process ⇒ Income Distribution
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Galor and Zeira (1988, 1993)

Unlike the Neoclassical Paradigm

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Main assumptions:

- Credit market imperfections
 - Differences in the interest rates for borrowers and lenders

and either

• Fixed investment cost in education or in other individual-specific projects

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- Inequality affects occupational choices: skilled vs. unskilled workers or entrepreneurs vs. workers
 - Non-poor economies:
 - Inequality ⇒ under-investment in human capital (inv't projects) that is transmitted across generations ⇒ lower output growth in the short-run and in the long-run
 - Poor economies:
 - Inequality permits some investment in HC (inv't projects) and may thus promote output growth
- The human capital channel is consistent with evidence (Perotti (1996))

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Echoes the hypothesis of the CMI Approach

- Inequality is harmful for the growth process
 - Inequality \Longrightarrow political pressure for redistribution
 - ullet Higher (distortionary) taxation \Longrightarrow lower investment and slower economic growth

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 - Inequality ⇒ Socio-Political instability
 - Socio-Political instability ⇒ reduces the security of property rights
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Gender inequality is harmful for the growth process

Galor-Weil (AER 1996

- Gender inequality reduces the opportunity cost of raising children more than
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- ⇒ increases fertility
- ⇒ reduces human capital investment (quantity-quality trade-off)
- ⇒ lowers female labor force participation
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- Unifies the Classical and the Modern Paradigms
- Provides an intertemporal reconciliation between conflicting viewpoints about the effect of inequality on economic growth
- Underlines the role of inequality in triggering socio-political transition
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- The replacement of physical capital accumulation by human capital accumulation as a prime engine of economic growth has changed the qualitative impact of inequality on the process of development
- ullet Early stages of industrialization: physical capital accumulation is a main engine of growth \Longrightarrow
 - Inequality enhanced development by channeling resources towards individuals whose marginal propensity to save is higher
- Later stages of development: the return to human capital increases due to capital-skill complementarity and human capital became the prime engine of growth
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- Human capital accumulation
- Physical capital accumulation

Human Capital vs. Physical Capital Accumulation

- Human capital is embodied in humans
 - Physiological constraints subjects its accumulation at the individual level to diminishing returns
 - The accumulation of human capital would be larger if it would be widely distributed among individuals in society
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- **Inequality** is conducive for **physical capital** accumulation, as long as the marginal propensity to save rises with income
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Inequality and Growth in Different Stages of Development

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- Overlapping-Generations economy
- t = 0, 1, 2, 3, ...
- One good
- 3 factors:
 - $K \equiv \text{Physical capital}$
 - $L^s \equiv \text{Skilled Labor}$
 - $L^u \equiv \text{Unskilled Labor}$

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Total output produced

$$Y_t = Y_t^s + Y_t^u$$

Production in the skilled-intensive sector:

$$Y_t^s = F(K_t, L_t^s) \equiv L_t^s f(k_t); \qquad k_t \equiv K_t/L_t^s$$

$$Y_t^u = aL_t^u$$

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Inverse Demand for Factors

• Capital:

$$r_t = f'(k_t) \equiv r(k_t)$$

Skilled labor:

$$w_t^s = f(k_t) - f'(k_t)k_t \equiv w^s(k_t)$$

• Unskilled labor:

$$w_t^u = a \equiv w^u$$

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Small open economy

• World interest = r

$$r_t = r$$

$$k_t = f'^{-1}(r) \equiv k$$

$$w_t^s = w^s(k) \equiv w^s(k)$$



$$(r_t, w_t^s, w_t^u) = (r, w^s, w^u) \qquad \forall$$

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$$(r_t, w_t^s, w_t^u) = (r, w^s, w^u)$$
 $\forall t$

- Continuum of measure 1
- Each Individual has 1 parent and 1 child
- Identical in:

Preferences
Innate abilities

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Differ in:

Member of Generation t: Period of Life

- First period of life (Period *t*):
 - [invest in HC] or [work as unskilled]
- Second period of life (Period t + 1):
 - [work as unskilled / no inv't in HC] or [work as skilled / inv't in HC in 1st period]

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- Time endowment:
 - 1 units of time in each period
- Capital endowment:
 - $b_t \equiv$ capital inherited in 1^{st} period
- Preferences:

$$u^t = \alpha \ln c_{t+1} + (1 - \alpha) \ln b_{t+1}$$
 $\alpha \in (0, 1)$

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$$u^{t} = \alpha \ln c_{t+1} + (1 - \alpha) \ln b_{t+1} \qquad \qquad \alpha \in (0, 1)$$

- Time endowment:
 - 1 units of time in each period
- Capital endowment:
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Second period budget constraint:

$$c_{t+1} + b_{t+1} \le \omega_{t+1}$$

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$$\{c_{t+1},b_{t+1}\}=\max[lpha\ln c_{t+1}+(1-lpha)\ln b_{t+1}]$$
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$$b_{t+1} = (1 - \alpha)\omega_{t+1}$$

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Indirect Utility: =>

$$v^t = \alpha \ln \alpha \omega_{t+1} + (1 - \alpha) \ln \omega_{t+1}$$

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- $\implies v^t$ is monotonic increasing in 2nd period wealth, ω_{t+1}
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Fundamental Assumptions

• Imperfect Capital Markets:

$$r < i$$
 (A1)

 $r \equiv$ interest rate for lender $i \equiv$ interest rate for borrowers (for inv't in HC)

Fixed cost of education (Indivisibility of inv't in HC) Weighted average of the
payments to teachers, administrators, and maintenance workers in the school system (i.e., weighted average of the
wages skilled and unskilled workers):

$$C^{H} = \theta w^{s} + (1 - \theta)w^{u} \equiv h > 0 \qquad \theta \in [0, 1]$$
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Income: Unskilled Individuals

$$\omega_{t+1}^u = (w^u + b_t)(1+r) + w^t$$

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 \Longrightarrow

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Assumptions

 Investment in human capital is not beneficial for individuals who must finance the entire cost of education via borrowing

$$w^s - (1+i)h < 0 \tag{A3}$$

• Investment in human capital is beneficial for individuals who can finance the entire cost of education *without* borrowing

$$w^{s} - (1+r)h > w^{u}(2+r)$$
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Income from Being Unskilled Worker

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Income from Being Skilled Worker: Lenders

$$b_t \quad \left\{ \begin{array}{l} < f \quad \rightarrow x^u_{t+1} > x^s_{t+1} \; (\text{individual } t \; \text{becomes unskilled}) \\ \\ > f \quad \rightarrow x^u_{t+1} < x^s_{t+1} \; (\text{individual } t \; \text{becomes skilled}) \end{array} \right.$$

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$$f = \frac{w^{u}(2+r) - [w^{s} - (1+i)h]}{i - r} > 0$$

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Bequest Dynamics

$$b_{t+1} = (1 - \alpha)x_{t+1}$$

$$b_{t+1} = \begin{cases} (1-\alpha)[w^u(2+r) + (1+r)b_t] & b_t \in [0, f] \\ (1-\alpha)[w^s - (1+i)h + (1+i)b_t] & b_t \in [f, h] \\ (1-\alpha)[w^s - (1+r)h + (1+r)b_t] & b_t \in [h, \infty) \end{cases}$$

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Bequest Dynamics: Sufficiet Conditions for Multiplicity of Steady-Sate

$$(1-\alpha)(1+r) < 1$$
 (A5) $(1-\alpha)(1+i) > 1$

$$(1-\alpha)w^s > h \tag{A6}$$

Bequest Dynamics

Bequest Dynamics: Multiple Steady-State Equilibrium

Bequest Dynamics: Stability of High Bequest Equilibrium

Bequest Dynamics: Stability of Steady- State Equilibria

The Distribution of the Inheritance in Period t

Income Distribution and the Long Run Decomposition of the Labor Force

$$\xi_t(b_t) \equiv {\sf Distribution}$$
 of inheritance at time t

$$\Longrightarrow$$

$$L_t = \int_0^\infty \xi(b_t) db_t \equiv 1$$

The Distribution of the Inheritance in Period t

Income Distribution of the Long Run Decomposition of the Labor Force

$$\lim_{t o \infty} I^u_t = \int_0^g \xi_t(b_t) db_t \equiv ar{I}^u$$

$$\lim_{t o\infty} I_t^s = \int_g^\infty \xi_t(b_t) db_t \equiv ar{I}^s$$

where

$$\partial \bar{I}^s/\partial g < 0$$

and

$$g = \frac{(1-\alpha)[(1+i)h - w^s]}{(1-\alpha)(1+i) - 1} > 0$$

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Income Distribution of Skill Composition

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 Income of a skilled individual in the second period of life (wage and capital income)

$$I_2^s = w^s + (\bar{b}^s - h)r$$

 Income of an unskilled individual in the second period of life (wage and capital income)

$$J_2^u = w^u + (\bar{b}^u + w^u)r$$

Income of an unskilled individual in the first period of life (only wage income)

$$I_1^u = w^u$$

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Aggregate income in the steady-state

$$\bar{Y} = I_2^s \bar{I}^s + I_2^u \bar{I}^u + I_1^u \bar{I}^u$$

• Aggregate income (note: $\bar{l}^s + \bar{l}^u = 1$)

$$Y = [w^{3} - rh + rb^{3}]I^{3} + [w^{a}(2+r) + rb^{a}](1-I^{3})$$

$$= w^{u}(2+r) + r\bar{b}^{u} + [(w^{s} - rh) - w^{u}(2+r) + (\bar{b}^{s} - \bar{b}^{u})]\bar{I}^{s}$$

Income per capita

$$\bar{y} = \bar{Y}/2$$

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Skill Composition and Income Per Capita in the Long Run

 An increase in the fraction of skilled workers increases income per capita in the steady-state

$$\frac{\partial \bar{y}}{\partial \bar{I}^s} = [(w^s - rh) - w^u(2+r) + (\bar{b}^s - \bar{b}^u)]/2 > 0$$

since

$$w^s - (1+r)h > w^u(2+r)$$

• An increase in g reduces income per capita in the steady-state

$$\frac{\partial \bar{y}}{\partial g} = \frac{\partial \bar{y}}{\partial \bar{l}^s} \frac{\partial \bar{l}^s}{\partial g} < 0$$

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Inequality and Development: Rich Economies

Rich Economies: Inequality is Harmful for Development

Inequality reduces human capital formation

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Inequality and Development: Poor Economies

Poor Economies: Inequality may Benefit Development

Inequality stimulates human capital formation

Poor Economies: Inequality may Benefit Development

- Education cost that is indexed to wages
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Robustness: Technological Progress and Endogenous Education Cost

Labor Augmenting Technological Progress: increases the productivity of workers in both the skilled-intensive and the unskilled intensive sector.

Production in the skilled-intensive sector

$$Y_t^s = F(K_t, A_t L_t^s) \equiv A_t L_t^s f(k_t); \qquad k_t \equiv K_t / A_t L_t^s$$

Production in the unskilled-intensive sector

$$Y_t^u = A_t a L_t^u$$

Technological progress

$$A_{t+1} = (1+\lambda)A_t$$

$$\lambda > 0$$
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Factor Prices

$$w_t^s = A_t[f(k) - f'(k)k] \equiv A_t w^s$$
 $w_t^u = A_t a \equiv A_t w^u$
 $r_t = r$

Cost of Education

- Weighted average of the payments to teachers, administrators, and maintenance workers in the school system
- ⇒ Weighted average of the wages skilled and unskilled workers

$$C_t^H = \theta A_t w^s + (1 - \theta) A_t w^u \equiv A_t H$$

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Threshold level of Bequest for Becoming Skilled Worker in Period t

$$f = \frac{A_t\{w^u(2+r) - [w^s - (1+i)h] - \lambda(w^s - w^u)\}}{(i-r)}$$

$$\frac{f_t}{A_t} = \frac{A_t \{ w^u(2+r) - [w^s - (1+i)h] - \lambda(w^s - w^u) \}}{(i-r)} \equiv \hat{f} > 0$$

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Bequest Dynamics

$$b_{t+1} = \begin{cases} (1-\alpha)\{A_t w^u(2+r+\lambda) + (1+r)b_t\} & b_t \in [0,f] \\ (1-\alpha)\{A_t [w^s(1+\lambda) - (1+i)h] + (1+i)b_t\} & b_t \in [f,A_t h] \\ (1-\alpha)\{A_t [w^s(1+\lambda) - (1+r)h] + (1+r)b_t\} & b_t \in [A_t h, \infty] \end{cases}$$

Bequest Dynamics

Let
$$\hat{b}_{t+1} \equiv b_{t+1} A_{t+1}$$

$$\hat{b}_{t+1} = \begin{cases} \left[\frac{1-\alpha}{1+\lambda}\right] \{w^{u}(2+r+\lambda) + (1+r)\hat{b}_{t}\} & \hat{b}_{t} \in [0,(\hat{f})] \\ \left[\frac{1-\alpha}{1+\lambda}\right] \{[w^{s}(1+\lambda) - (1+i)h] + (1+i)\hat{b}_{t}\} & \hat{b}_{t} \in [\hat{f},h] \\ \left[\frac{1-\alpha}{1+\lambda}\right] \{[w^{s}(1+\lambda) - (1+r)h] + (1+r)\hat{b}_{t}\} & \hat{b}_{t} \in [h,\infty] \end{cases}$$

 \Rightarrow The dynamical system is unaffected qualitatively by labor-augmenting technological progress

Bequest Dynamics

Let
$$\hat{b}_{t+1} \equiv b_{t+1} A_{t+1}$$

$$\hat{b}_{t+1} = \begin{cases} \begin{bmatrix} \frac{1-\alpha}{1+\lambda} \end{bmatrix} \{ w^u(2+r+\lambda) + (1+r)\hat{b}_t \} & \hat{b}_t \in [0,(\hat{f})] \\ \\ \begin{bmatrix} \frac{1-\alpha}{1+\lambda} \end{bmatrix} \{ [w^s(1+\lambda) - (1+i)h] + (1+i)\hat{b}_t \} & \hat{b}_t \in [\hat{f},h] \\ \\ \\ \begin{bmatrix} \frac{1-\alpha}{1+\lambda} \end{bmatrix} \{ [w^s(1+\lambda) - (1+r)h] + (1+r)\hat{b}_t \} & \hat{b}_t \in [h,\infty] \end{cases}$$

 \Rightarrow The dynamical system is unaffected qualitatively by labor-augmenting technological progress

Sufficient Conditions for Multiple Steady-States

$$(1 - \alpha)(1 + r) < (1 + \lambda)$$

 $(1 - \alpha)(1 + i) > (1 + \lambda)$
 $w^{s}(1 + \lambda) - (1 + i)h < 0$

 \Rightarrow The system is characterized by multiple steady-state, where the unstable equilibrium

$$\hat{g} = \frac{(1-\alpha)[(1+i)h - w^s(1+\lambda)]}{[(1-\alpha)(i+i) - (1+\lambda)]} > 0$$